A NEW ENTRANT'S DECISION ON VIRTUAL VERSUS BRICKS-AND-MORTAR RETAILING

Gilles Reinhardt
Department of Management
DePaul University
greinhar@condor.depaul.edu

Moren Lévesque
Department of Marketing and Policy Studies
Case Western Reserve University
mxl101@cwru.edu

ABSTRACT

Firms retail an increasing proportion of their products on-line. How much to sell through the Internet is a decision driven by market consideration and supply chain efficiency. In this paper, we use micro-economics to derive a firm's on-line and off-line quantity that best trade-offs costs, revenue, and competitive behavior. We study the case of a new firm which is a monopolist. We show that it may not be optimal for the firm to retail on both channels. Indeed, the firm must consider all costs in retailing its products. The model is then refined to study the case of a new entrant facing an already populated market that operates on one or both channels. To maximize profits it may be tempting for the new entrant to retail on-line if the incumbents are retailing on a bricks-and-mortar network, or vice-versa. We show that this decision depends almost solely on a product's typology and the firm's supply chain efficiency. We also observe a new competitive strategy in the proportion of output one firm sells on-line: as that proportion increases for the existing firms, it will decrease for the new entrant, and vice versa.

Keywords: E-commerce, retail channels, supply chain management, optimization, micro-economics

1. Introduction

Motivated by the potential of the Internet, firms retail an increasing proportion of their products on-line. The decision of how much (if any) to sell through the Internet is essentially driven by market consideration and supply chain efficiency. In this paper, we use a simple micro-economic model to derive, under a set of regularity assumptions, the firm's on-line and off-line quantity that best trade-offs costs, revenue, and competitive behavior.

We begin by studying the case of a new firm ("entrant") which enters a market as a monopolist (with a *radical* innovation; e.g. a value proposition not yet available on the market). The firm sees potential revenues from a price-sensitive market demand on both a bricks-and-mortar and an on-line channel. We show that this in itself is insufficient to induce a firm to retail on both channels. Indeed, the firm must consider supply chain costs in retailing its products, namely manufacturing, production, inventory, and shipping. Many entrants on the market have had serious difficulty turning a profit, mostly due to the induced complexity of managing production, inventory, and distribution when operating on both channels (Chopra and Van Mieghem, 2000). Failing to distinguish and properly manage its costs on each channel has proved at least challenging, if not fatal to many firms.

The model is then refined to study the case of an entrant (with an *incremental* innovation) facing an already populated market that operates on one or both channels. Putting aside competition amongst the existing firms in the market, we model the entrant as a new firm competing against a group of existing firms, which we collapse into a single monopoly. Doing so permits the derivation of useful and sometimes counter-intuitive results. For example, to maximize profits it may be tempting for the entrant to retail on-line if the incumbents are retailing on a bricks-and-mortar network, or vice-versa. We show that this decision depends almost solely on a product's typology and the firm's supply chain efficiency. We also observe a new competitive strategy in the proportion of output one firm sells on-line: as that proportion increases for the existing firms, it will decrease for the new entrant, and vice versa.

We anticipate our contribution from this analysis to be manifold. First, a distinctive product typology (digitalizable vs solid) which is exclusively defined by the supply chain costs each type incurs when being retailed on-line or off-line. Second, a contribution to the e-commerce literature by couching the problem in an analytical framework that explicitly models the dynamics of sales revenues against supply chain costs for a firm that operates on two distinct retailing channels, on- and off-line. Third, the solutions derived from our model unearth profit maximizing decisions and competitive strategies that more traditional approaches may overlook, again chiefly due to the impact on the new entrant's bottom line of entering or increasing its market via the Internet. Fourth, we target our analysis towards two distinct classes of entrants: those with a radical innovation (they enter the market as monopolists) and those with an incremental one (they enter an already populated market).

The remainder of this paper is organized as follows. Section 2 briefly reviews related literature whereas section 3 offers a product typology. Our results are derived under a set of costs, market and model based assumptions, which we discussed in section 4. Section 5 presents the monopoly results and section 6 studies analogous questions in a duopoly setting. Section 7 concludes the paper and specifies directions for further research. Technical details appear in an appendix.

2. Related literature

When a new entrant is making the virtual/bricks-and-mortar decision, it must consider a number of issues including consumer preferences, channel objectives and constraints, supply chain structure, and overall value creation for the firm. New academic research on the e-commerce phenomenon has emerged from most fields of management such as marketing, strategy, information, and operations (Hoffman, 2000; Anupindi, Bassok and Zemel, 2001). Numerous studies have investigated the role of a virtual existence on consumer shopping behavior. Messinger and Narasimhan (1997) investigate why and how some market segments prefer one retail channel over others, while Keeney (1999) identifies the value that Internet commerce brings to the customer. Kaufman-Scarborough and Lindquist (2002) analyze how consumers use and integrate the multi-channel options in their shopping experience by testing how the frequency of on-line browsing or shopping relates to various flexibility attributes provided by this channel. Schoenbachler and Gordon (2002) propose a model of multi-channel buyer behavior to tackle the challenge brought about by the multiplicity of channels the firm faces and suggest a "consumer-centric view rather than a channel focused view to work through the challenges unique to the multi-channel marketer." These are consumer focused papers that analyze the impact of digitalization on the shopping experience.

Research has also shown to new entrants the importance of channel objectives on making the channel choice. Balasubramanian (1998) offers Salop's (1979) circular spatial market to capture the competitive trade-offs between a direct marketer and *N* retailers. The foci of that paper are the role of store location as a competitive advantage, the impact on existing bricks-and-mortar locations on a direct marketer entrant, and the role of product information as a competitive lever. Tsay and Agrawal (2001) investigate channel conflicts between a manufacturer and a reseller via a stylized supply chain with independent decision making at both the manufacturer and reseller levels. Vishwanath and Mulvin (2001) provide evidence that *pure-players* (firms that exclusively retail on-line) are gradually being outperformed by retailers who exploit more than one channel, but who do so as complements to one another (by defining distinct roles for each channel and by leveraging on- vs off-line synergies). They state that "less than 5% of pure-players are profitable today, and many are in dire straits." Levin, Levin, and Heath (2003) also study how a retailer can best utilize multiple channels. Their study measures the preferences of consumers for retailing channels as they progress through the shopping experience (searching, comparing, then purchasing). They also test consumer's reactions to strategic alliances amongst retailers of on-line and off-line brands.

While some have gone beyond a consumer focus and modeled an overall value creation potential of e-business (e.g., Bakos, 2001; Devinney, Latukefu and Midgley, 2001; Lucking-Reiley and Spulber, 2001) or the effects of direct sales on firms, consumers and welfare altogether (e.g., Hendershott and Zhang, 2001), other scholars have taken the firm's perspective rather than the consumer's. These include Hitt and Frei (2002) who compare the value to the firm of banking customers who use on-line services rather than the bricks-and-mortar locations. Leamer and Storper (2001) have looked at the role of the Internet in increasing the fitness of the division of labor and automating intermediation and coordination tasks, whereas Dewan, Jing and Seidmann (2000) have examined sellers' costs reduction for collecting buyer preference information and managing multiple prices. Others have looked at bricks-and-clicks integration (e.g., Gulati and Garino, 2000) and competitive implications of bundling information goods (Bakos and Brynjolfsson, 2000).

Our analysis addresses how an incoming firm's choice of retail channel, specifically off- vs on-line, impacts its operational profitability and competitive advantages. Our model studies these impacts by first partitioning the firm's cost function into supply chain components (production, inventory & distribution, retailing) in order to directly incorporate into its supply chain the cost impact of a product's digital content and, second, by integrating total output and proportion distributed on- and off-line as decision variables. We thus model a "new economy" decision that most retailers must now face.

3. Product typology

Figure 1 describes a generic supply chain that illustrates the flow of goods through a bricks-and-mortar network. Figure 2 describes the distribution of the same product but through an on-line retailer's supply chain. Firms satisfy their customer's orders directly from inventory and in effect disintermediate their retailing activity (which they conduct either through their own network or by selling to existing retailing chains). We term *digitalizable* a product for which the inventory and distribution costs decrease as the proportion of output retailed on-line increases. Analogously, we term *solid* a product for which the inventory & distribution cost is increasing in that proportion. Software, data, newspapers, music, airline tickets, and financial transactions can all take digital form. We classify such products as digitalizable as they can be stored, distributed and retailed at very low variable costs (file transfers & credit card transactions). A solid product such as a computer, a book, a toy, clothes and groceries incurs higher inventory and distribution costs since it requires storage, handling, and delivery, irrespective of its retailing channel. This digitalizable-solid typology differs from the "high touch" and "low touch" distinction from the literature (The Economist, 2000, Levin, Levin, and Heath, 2003) in that it focuses on supply chain issues and not consumer interactions with the product being offered.[1]

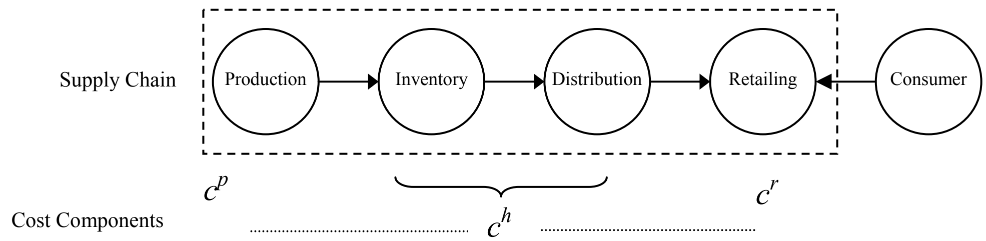

Figure 1: Supply chain for a bricks-and-mortar network

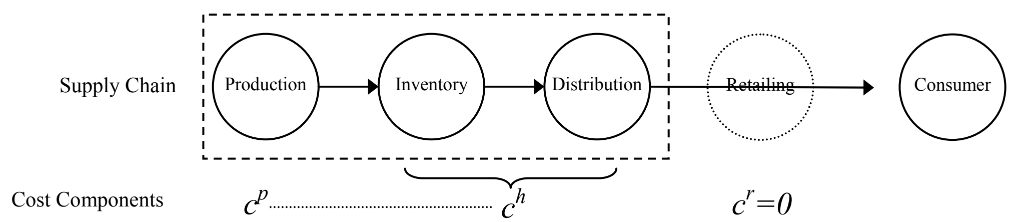

Figure 2: Supply chain for a virtual store network

"E-tailers" have thus had to implement innovative supply chain management techniques such as tailoring a new inventory and distribution system specifically for on-line retailing, establishing large scale contracts with third party shippers, or launch strategic alliances with bricks-and-mortar outlets. Conversely, high touch goods such as specialized application software or computer games are quite expedient for on-line retailing since they can seamlessly be delivered to a customer's desktop and can be "test driven" before being purchased (for example, by releasing evaluation versions or limiting functionality until the customer buys the product). "Cripple-ware" and time-limited demos are now commonly available, as they help consumers to conduct their own research on comparisons, capabilities, and reliabilities before buying such software.

Although digitalizable products appropriate themselves quite well to on-line retailing, we show that solid products may not. For a monopolist we demonstrate that *interior* solutions are rare (where, by interior, we mean solutions where the firm distributes both on-line and off-line). A *corner* solution is one where the firm distributes on

[1] A product is deemed "high touch" if it is easier to sell when the customer is given the opportunity to closely interact with it before making the purchase decision whereas a "low touch" good is more customary and requires less interaction with the customer before the purchasing decision. The low touch attribute tends to make on-line retailing appropriate since the customer requires little interaction with the good before making the purchase. Yet books, often seen as low touch goods, are not necessarily expedient for profitable on-line retailing since the functionality and cost effectiveness of the Internet cannot be harnessed to reduce their inventory & distribution costs.

only one channel. The existence of corner or interior solutions in our model depends *solely* on the cost structure of the supply chain, particularly on the direction and shape of the inventory & distribution and retailing cost components. Since our definition of digitalizable and solid goods is entirely based on the supply chain cost they incur, our model does not take into account the ancillary benefits of operating on both channels, such as offering to Internet customers the opportunity to browse and compare attributes, or providing Bricks-and-Mortar customers the opportunity to go on-line inside the location and customize their orders.

In a special duopoly case, the leader-follower model, we show that both firms should retail a significant proportion of output on-line for digitalizable products. We derive conditions for corner and interior solutions and note that interior equilibria (in which competitors simultaneously sell on- and off-line) are quite restricted. Their existence depends again on each firm's cost structure.

4. Cost structure

4.1. Cost description

We assume a three-tiered cost function

$$c(Q,f) = c^p(Q,f) + c^h(Q,f) + c^r(Q,f), \tag{1}$$

where Q is the firm's total output and f is the firm's on-line strategy, i.e. the proportion of that output supplied on-line (1-f is the proportion supplied off-line). We let $c^x(f \mid Q)$ be the cost components, given output Q, where $x = p, h,$ or r are as defined in Figures 1, 2 and Equation 1. Note that Equation 1 excludes *marketing* costs, the cost of establishing and maintaining a product's market share through an on- or off-line channel. We weigh our focus towards supply chain costs as those often dictate a firm's profitability when more than one channel is available (Chopra and Van Mieghem, 2000). Marketing costs, of course, play a key role in retailing channel success. Schoenbachler and Gordon (2002) indicate that "Web only retailers spend nearly double ($42 vs $22) the amount to acquire a new customer than multi-channel marketers spend on customer acquisition. Multi-channel players, though, spend over five times what Internet only marketers spend on customer retention (Jaffe, 2000)." They also state that multi-channel players spend significantly less for marketing and advertising than pure-players, in spite of the sizeable cost of creating a Web presence which ranges from $1.5 million to $3 million.

Our model, however, is only minimally affected by this egregious discrepancy in marketing cost between pure and multi-channel players. Its main primitive, as previously specified, is a new *entrant* who creates a market (monopoly model) by setting a profit-maximizing price, or who enters a populated one (competitive model) and sets an equilibrium price. In both settings, they face a market characterized by a demand curve which we assume independent of the firm's actions to increase recognition or spur sales (either are consequences of increasing marketing costs). The results derived from our model could possibly carry over to a setting where marketing costs are included but the solutions would then exhibit such added complexity that, although more precise, would shield the insights our results add to the perspective of supply chain efficiency.

c^p, the production costs, include factor input, other capital, and labor. Moreover, we assume c^p to be the purchasing cost of a product if the firm does not manufacture in-house. c^h, the inventory & distribution costs, cover the packaging and storage of finished goods, the maintenance of said storage, the retrieval of goods in inventory, and their delivery. We explicitly model the delivery charge as a cost to the firm, even if it is often transferred to the customer. c^r, the retailing costs, include the cost to own, rent, manage, or access shelf space in shopping areas. We set $c^r = 0$ for firms that retail exclusively on an on-line channel.

4.2. Cost rate of returns: A formal definition for product typology

At no loss of generality in our results, we assume each cost component differentiable in both output Q and on-line strategy f. In Table 1, we make additional first order assumptions on the cost function that allow us to formally define the notion of a digitalizable product as follows:

DEFINITION: We term *digitalizable* a product for which both the inventory & distribution (c^h) and retailing (c^r) cost components are increasing in Q *and* decreasing in f given Q. Analogously, we term *solid* a product for which the inventory & distribution cost is increasing in Q and in f given Q, and for which the retailing cost is increasing in Q but decreasing in f given Q. In our model, the difference between solid and digitalizable lies solely in the first order behavior of the inventory & distribution cost in f given Q.

The cost of production is assumed increasing in Q, but constant in f given Q since the firm's choice of retailing channel has no effect on this cost. We note that for digitalizable products production costs for additional units beyond the first one are still increasing but at a slower pace than for solid goods. For solid products, inventory & distribution costs (in f given Q) do suffer when the number of delivery points increases or when orders increase in number but decrease in size. The inventory systems are more complex since on-line orders for a specific item rarely exceed one unit (e.g. a customer hardly ever orders more than one copy of a book). An on-line retailer of solid goods must also ensure that the purchases reach the individual customers, generally through a mail or courier service. Even if borne by the customers the distribution costs entailed often prevent the on-line retailer from pricing competitively against a bricks-and-mortar retailer. For digitalizable products however, that cost decreases since only one unit needs to be stored and copies can be made and shipped electronically on request and payment. In fact, $c^h(f|Q)$ being increasing for solid goods and decreasing for digitalizable goods is the *only* cost component that does not behave the same for solid versus digitalizable goods. The cost of retailing is assumed to decrease in f given Q since the firm does not need to access as many bricks-and-mortar outlets to achieve the same sales level.

Table 1: Assumptions on first order behaviors (rate of returns) of cost components

Cost Component	Solid Product		Digitalizable Product	
	in Q	*in f given Q*	*in Q*	*in f given Q*
Production: c^p	increasing	Constant	increasing	constant
Inventory & Distribution: c^h	increasing	*Increasing*	increasing	*decreasing*
Retailing: c^r	increasing	Decreasing	increasing	decreasing

Knowledge of the second order behavior of the cost function is key to characterizing a firm's on-line strategy. For example, consider an exclusive off-line retailer of a digitalizable good. By our definition of digitalizable, $c_f(f|Q) < 0$, i.e. the derivative (rate of return) of the total cost function with respect to f (given Q) is negative. Now suppose the firm begins retailing this good on-line. Its total costs either decrease at an increasing rate (concave c) or decreasing rate (convex c). In the latter case, the firm captures the bulk of the savings from moving on-line immediately: savings from shutting down bricks-and-mortar outlets (in the c^r portion) outweigh the increased costs induced by the new complexity of managing inventory & distribution at the customer level (the c^h portion), and this gap decreases as the on-line presence, f, increases.

5. Monopoly Results

We solve the problem for the on-line strategy f given firm output Q for a new firm with a radical innovation entering a market as a monopolist. Findings are for an economy with one and two markets.

5.1. One market

The monopolist's problem is to find the value of f that maximizes total profits, i.e.

$$\operatorname*{Max}_{f} \left(A-Q\right)\times Q \; - \left(c^{p}(f\,|\,Q)+c^{h}(f\,|\,Q)+c^{r}(f\,|\,Q)\right), \tag{2}$$

where A-Q is the (linear) demand curve that the monopolist faces. A is the (positive) intercept on the price axis, and $(A\text{-}Q)\times Q$ is the total revenue that the firm generates. Table 2 presents the on-line strategies under a fixed output level (i.e. f^* given Q) that solve Equation 2. Since we assume linear demand, the optimal on-line strategy only depends on the behavior of the total cost function, $c^{p}(f\,|\,Q)+c^{h}(f\,|\,Q)+c^{r}(f\,|\,Q)$. Formally,

PRESCRIPTION 1:

(i) If its product is digitalizable, then the sum of inventory & distribution costs and retailing costs, $c^h(f|Q) + c^r(f|Q)$, is decreasing and the monopolist only retails on-line (Table 2 cases 2 and 5).

(ii) If its product is solid, a profit maximizing monopolist only retails on one channel whenever the sum of its inventory & distribution costs and retailing costs is either

(ii.a) non-monotone and has non-increasing returns (i.e. weakly concave: Table 2 case 3), or

(ii.b) monotone (Table 2, cases 1, 2, 4, or 5). Note that cases 2 and 5 apply to both solid and digitalizable products because the sum of a firm inventory & distribution and retailing costs could also be decreasing for solid products.

(iii) If its product is solid, a profit maximizing monopolist should retail both on- and off-line whenever the sum of its inventory & distribution costs and retailing costs is non-monotone and has increasing returns (i.e. convex: Table 2 case 6).[2]

Table 2: Optimal on-line strategy (f^* given Q)

Inventory/Distribution and Retailing Costs in f given Q: $c^h(f\|Q) + c^r(f\|Q)$	f^*
Case 1: Concave increasing	0
Case 2: Concave decreasing	1
Case 3: Concave non-monotone	0 if $c^h(1 \|Q) + c^r(1 \|Q) \geq c^h(0 \|Q) + c^r(0 \|Q)$ 1 if $c^h(1 \|Q) + c^r(1 \|Q) < c^h(0 \|Q) + c^r(0 \|Q)$
Case 4: Convex increasing	0
Case 5: Convex decreasing	1
Case 6: Convex non-monotone	$f^* \in (0,1)$ at $\frac{dc^h(f \mid Q)}{df} + \frac{dc^r(f \mid Q)}{df} = 0$

In (i) off-line retailing is not attractive because both inventory & distribution and retailing costs decrease monotonically in the on-line strategy f. This result explains the many businesses that, although operating in competitive environments, have shifted retailing of their digitalizable goods on-line (financial institutions, transportation & accommodation suppliers, software and data retailers).

In (ii) the firm only retails off-line ($f^* = 0$) when the savings in retailing costs do not outweigh the additional costs in inventory & distribution, i.e. $c^h + c^r$ is increasing for all $f \in [0,1]$. However the firm only retails on-line ($f^* = 1$) if $c^h + c^r$ is decreasing for all $f \in [0,1]$. This prescription proposes a justification to the soaring successes of on-line retailers of digitalizable goods, but also to the challenge at turning a profit for many on-line retailers of solid goods. To turn profitable the activity, a monopolist should exhibit a lower c^h (inventory & distribution cost) than its opportunity savings of not operating bricks-and-mortar. These results may not be observable when objectives other than profit maximization (i.e. stock valuation, market penetration, product or firm position) dictate the on-line strategy. Observe from Figure 3 that, although not operating in monopoly settings, industries retailing a solid, high touch good exhibit forecasted on-line sales of no more than 10% of total. Nonetheless, the popular e-commerce sites are typically retailers of solid goods (The Economist, 2000) and most are operating at losses. This has induced on-line retailers to adopt state-of-the-art inventory and distribution systems (locations, stocking/picking mechanisms, strategic alliances, and efficient contracts with third party shippers).

In (iii) the firm sets its on-line strategy exactly where the rate of savings from shutting down one retail outlet matches the increased expenses from the additional delivery points. Many successful retailers of solid products have indeed opened an on-line retailing channel but have also kept their bricks-and-mortar presence. Thus, one possible avenue for an entrant monopolist who sells, say, a new home electronic device would be to retail it on both on- and off-line channels but keep their fixed costs low, perhaps by selling through intermediates such as Amazon and Circuit City.

[2] The proportion of on-line supply is given by $f^* \in (0,1)$ at $\frac{dc^h(f \mid Q)}{df} + \frac{dc^r(f \mid Q)}{df} = 0$.

5.2. Two markets

The open-to-the-world nature of the Internet suggests that demand for a product available on-line dominates off-line demand because the potential reach of the on-line market is global, and that many customers would not be reached without an on-line market. This belief has arguably contributed to the "dot com effect" (soaring values of initial public offerings of Internet retailers). Many "e-tailers" expand their on-line offerings, and do so based on the tenet that on-line potential demand dominates off-line. Our model attempts to predict on- and off-line profitability of products based on their digital content level. We thus include this tenet in our model to tie our results with this belief.

Specifically, we assume that the firm simultaneously faces two markets characterized by the following demand curves: $P = A_I - bQ_I$ for on-line retailing and $P = A_B - Q_B$ for off-line retailing, where $A_I > A_B > 0$ and $0 \le b \le A_I / A_B$. Similarly to the description in Section 5.1, $A_I - bQ_I$ is the (linear) on-line demand curve with A_I as the (positive) on-line intercept on the price axis and b the slope of the demand curve. $A_B - Q_B$ is the off-line demand curve (with a standardized slope equal to 1), and A_B is the (positive) off-line intercept on the price axis. The two conditions, $A_I > A_B > 0$ and $0 \le b \le A_I / A_B$, ensure that the on-line demand curve dominates the off-line one. We plot a representation of the demand curves in Figure 3.

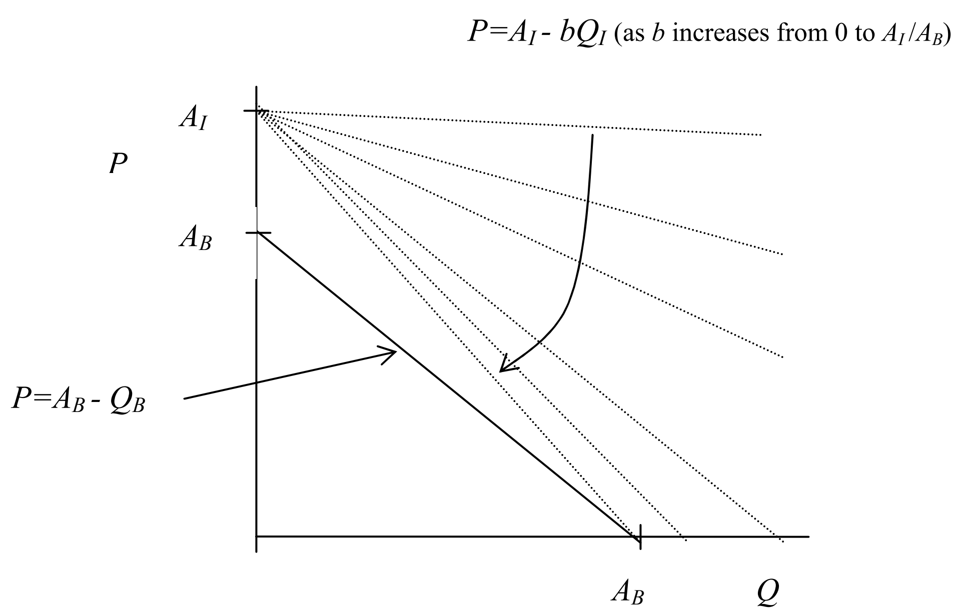

Figure 3: Demand curves faced by the firm

The monopolist's problem is now

$$\underset{f}{\text{Max}}\, \left(A_I - bfQ\right)\times fQ \;+\; \left(A_B - (1-f)Q\right)\times (1-f)Q \;-\; c(Q,f)\,, \tag{3}$$

where $\left(A_I - bfQ\right)\times fQ$ is the total revenue the firm generates from the on-line market, whereas $\left(A_B - (1-f)Q\right)\times (1-f)Q$ is that from the off-line market. Total cost $c(Q,f)$ is given by Equation 1. Existence of interior solutions depends on the convexity (or absence thereof) in the cost function, and on the on-line and off-line demand profiles ($A_I - A_B$, b) of the value proposition. Let c_f be the partial derivative of $c(f\,|Q)$ with respect to f. The first order (sufficient) condition for a maximum is

$$\frac{c_f - \left(A_I - A_B\right)Q}{2Q^2} = 1 - \left(1+b\right)f\,. \tag{4}$$

We distinguish two cases: $c(Q,f)$ linear in f, and $c(Q,f)$ convex (increasing returns) or concave (decreasing returns) in f. Formally, for the former case, PRESCRIPTION 2**:** Let total cost $c(Q,f)$ be linear in the on-line strategy f with a constant slope $c_f = \alpha$. For digitalizable goods $\alpha < 0$ and thus $f^* \geq 1/(1+b)$ or, equivalently, $f^* \in \left[1 - \dfrac{A_I}{A_I + A_B}, 1\right]$.[3] More generally,

(i) the firm should exclusively retail on-line ($f^* = 1$) whenever $\dfrac{\alpha - (A_I - A_B)Q}{2Q^2} \leq -b$;

(ii) the firm should retail off-line ($f^* = 0$) whenever $\dfrac{\alpha - (A_I - A_B)Q}{2Q^2} \geq 1$;[4] and

(iii) the firm should retail on both channels ($f^* = \dfrac{(A_I - A_B)Q + 2Q^2 - \alpha}{2(1+b)Q^2}$) whenever $-b \leq \dfrac{\alpha - (A_I - A_B)Q}{2Q^2} \leq 1$.

The first part of Prescription 2 calls for positive supply to the on-line channel if the firm retails a digitalizable good, but not necessarily exclusive supply since, in this model, there is distinct profit opportunity on the off-line channel. More generally, the monopolist's on-line strategy (given a fixed output Q) is represented in Figure 4. This strategy profile is reminiscent of the monopolists' optimal output level set where marginal revenue equals marginal cost. In this development, Q is fixed and α is the savings rate embodied in the firm's total cost function with respect to its on-line strategy. We observe that $(A_I - A_B)Q$ is the incremental revenue to the firm of supplying on-line. If the difference is too positive (negative), then the firm should exclusively retail off-line (respectively, on-line).

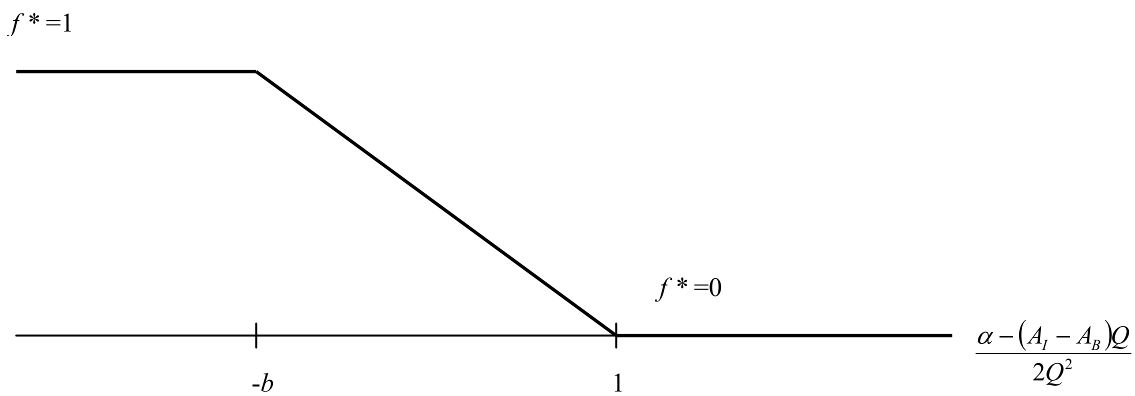

Figure 4: Monopolist's on-line strategy given a fixed output level Q and $c_f = \alpha$

When $c(Q,f)$ is strictly concave or convex in f, we perform our analysis using Figures 5 and 6. The optimal strategy profile is given by f^* which solves Equation 4. The optimal on-line strategy thus depends on the convexity (or concavity) of the inventory & distribution and retailing portions of the total cost function. For a digitalizable product, as Figure 5 shows, a firm may benefit from retailing both on a bricks-and-mortar network and on a virtual store if its cost function, despite being decreasing, does not capture enough savings from moving more output on-line (i.e. is not convex enough). Indeed, as the function's convexity decreases, the left-hand side of Equation 4 flattens and, for a fixed b, we note that the set of possibilities for a crossing that would yield $f^* < 1$ coarsens. For digitalizable goods, the left-hand side curves are both negative at $f=0$. They increase (decrease) if the cost function is convex (concave), but not necessarily linearly.

[3] This result directly follows from Equations 4 and $b \in [0, A_I / A_B]$.

[4] Although the right-hand side of Equation 4 cannot exceed 1, its left-hand side can.

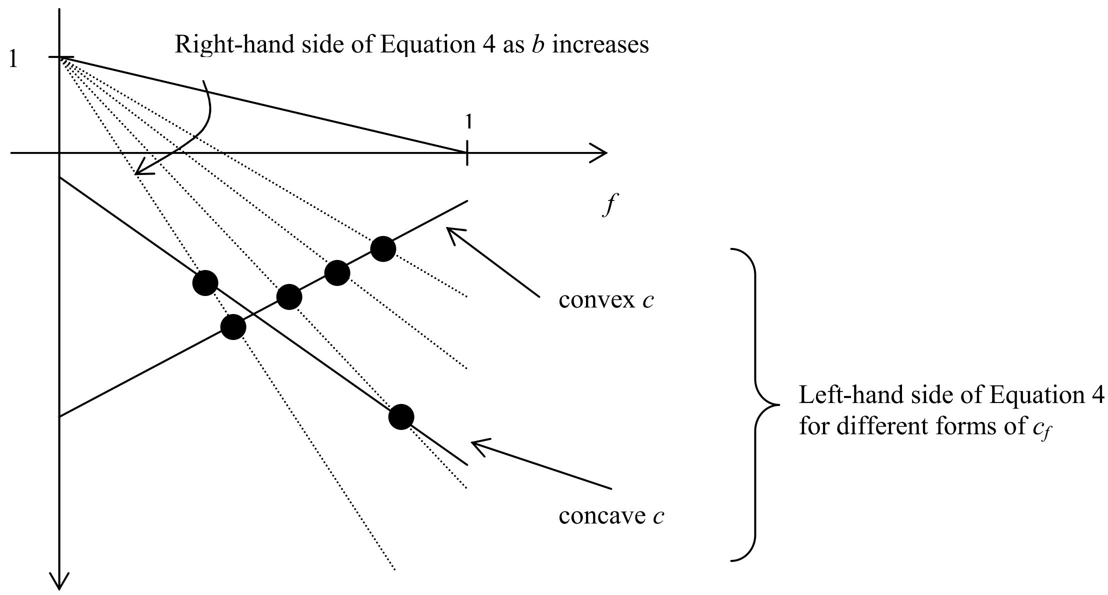

Figure 5: Characterization of f^* for **digitalizable** goods (given by the f-axis coordinate of the black dots)

We also note that digitalizable goods with more elastic on-line demand (small b) call again for a strong (if not exclusive) on-line presence since the set of possibilities for a crossing that would yield $f^* < 1$ coarsens again. Moreover, for a given cost structure, we observe that f^* decreases as b increases. Figure 6 shows that, *ceteris paribus*, the on-line strategy f^* will be less for a solid product than a digitalizable one. We first recall that a solid product need not exhibit $c_f < 0$ (see Table 1). To illustrate the effect on f^*, we simply redraw the f-axis at a lower position and observe that the intersection points all shift left on the horizontal axis. For solid goods, the left-hand side curves shift up compared to Figure 5. For a fixed b, we observe that f^* shift left for tangible goods, i.e. a larger proportion of output is retailed off-line.

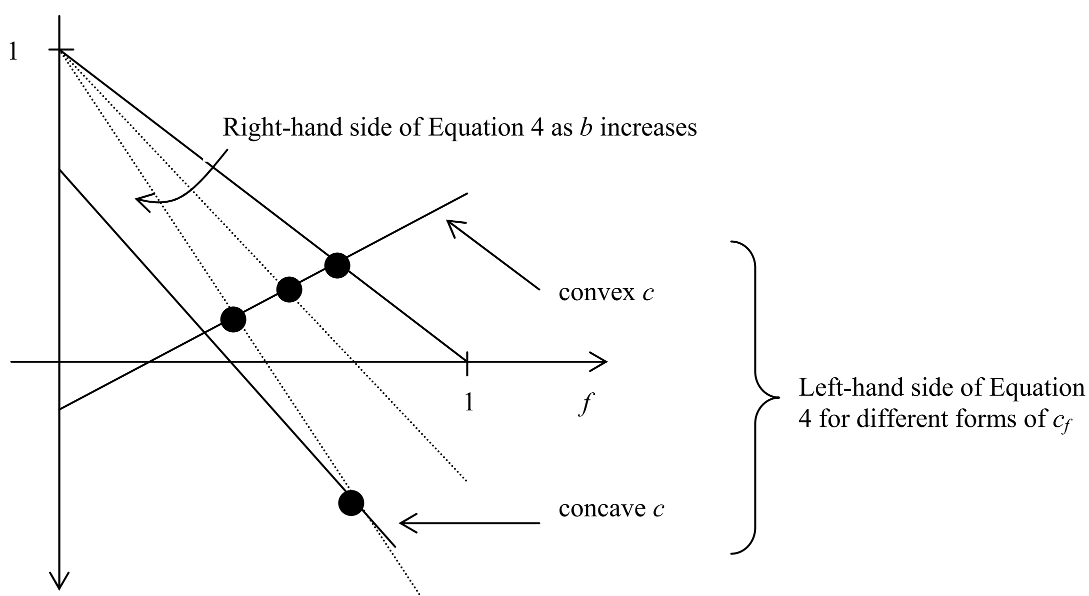

Figure 6: Characterization of f^* for **solid** goods (given by the f-axis coordinate of the black dots)

6. Duopoly Model

We now refine our model to include the scenario where the new firm enters an already populated market. A duopoly approach implies that the market only has an incumbent monopolist when the new firm enters. Hence, the results derived do not immediately apply to a general setting of multiple incumbents. Yet, the general prescriptions appropriate themselves to this general setting, if one puts aside the competitive behavior across the existing incumbents. We derive results under a "leader-follower" scenario: given an incumbent monopolist operating exclusively on- or off-line, where should a potential entrant to the industry position itself, i.e. how much of its output should be retailed on- and off-line? For example, a sizeable portion of the market share for computers has shifted to Dell and other on-line retailers. Compaq, the leader in off-line retailing for computers, reacted by opening its own on-line store. Similar dynamics can be observed in book retailing, banking, trading, travel services, and the like. Equilibrium conditions under which both firms achieve *maximum separation* (one on-line, one off-line), compete exclusively on one market, or on both are also derived for the leader-follower scenario. We now tailor the model assumptions and notation for a duopoly framework.

In a decoupled market setting, demand is given by

$$\begin{aligned} p_B &= A_B - Q_B = A_B - (1-f_i)q_i - (1-f_j)q_j \\ \text{and} \quad p_I &= A_I - bQ_I = A_I - bf_iq_i - bf_jq_j, \quad i,j = 1,2;\, i \neq j \end{aligned} \tag{5}$$

and the cost functions for each firm are given by

$$c^i(f_i \mid q_i) = c^{i,p}(f_i \mid q_i) + c^{i,h}(f_i \mid q_i) + c^{i,r}(f_i \mid q_i),\; c^i_{f_i} = \frac{dc^i(f_i \mid q_i)}{df_i},\; c^i_{f_if_i} = \frac{d^2c^i(f_i \mid q_i)}{df_i^2} \tag{6}$$

where we add the superscript i (equals to 1 or 2) to distinguish between the two firms. In this analysis c^2 always refers to firm 2's costs and *not* the square of c. We also make a distinction between market quantities and firm output with a capital Q for the former.

Firm 2 is assumed to be the leader and an incumbent monopolist operating on only one channel (off-line or on-line). Hence, we fix $f_2 = 0$ or 1 and $q_2 = \frac{A_B - c^2_{q_2}}{2}$ or $q_2 = \frac{A_I - c^2_{q_2}}{2b}$, respectively, where $c^2_{q_2}$ is firm 2's cost function derivative with respect to q_2. Firm 1 is an entrant to the industry and maximizes its profits given by

$$\Pi_1(f_1 \mid q_1; f_2, q_2) = (A_I - bf_1q_1 - bf_2q_2) \times f_1q_1 + (A_B - (1-f_1)q_1 - (1-f_2)q_2) \times (1-f_1)q_1 - c^1(f_1 \mid q_1), \tag{7}$$

where c^1 is firm 1's cost function given a fixed output level q_1. Assuming that c^1 has increasing returns (convex) in f_1, we derive that firm 1's optimal on-line strategy is the solution to

$$\frac{c^1_{f_1} - (A_I - A_B)q_1}{2q_1^2} + \frac{A_B - c^2_{q_2}}{4q_1} = 1 - (1+b)f_1 \tag{8}$$

if firm 2 operates exclusively off-line ($f_2 = 0$) and the solution to

$$\frac{c^1_{f_1} - (A_I - A_B)q_1}{2q_1^2} - \frac{A_I - c^2_{q_2}}{4q_1} = 1 - (1+b)f_1 \tag{9}$$

if firm 2 operates exclusively on-line ($f_2 = 1$).

Again we perform our analysis using the schematic illustrations in Figures 5, 6, 7a, and 7b. Comparing Equations 8 and 9 to Equation 4, we note that the left-hand side is larger (smaller) if firm 2 retails exclusively off- (respectively, on-) line.[5] The representative (left-hand side) curves in Figure 7a (respectively, b) will hence shift down (respectively, up), compared to the analogous curves in Figures 5 and 6. Hence, there indeed is a competitive advantage for an entrant to separate from the incumbent and choose a distinctively lower or higher on-line strategy. Figure 7a shows that the larger the leader's output is, the lower the left-hand side curve shifts. For a convex cost function, this means that Equation 8 is only satisfied for values of f_1^* that are strictly positive. Moreover, we observe that the range of solutions where $f_1^* = 1$ increases, i.e. extensions of the left- and right-hand side curves are

[5] We also note that this change is closely related to the leader's output since it increases by $(1/2q_1)\times(A_B - c^2_{q_2}/2)$, a multiplicative "constant" away from the leader's output (the standard monopoly profit maximizing quantity). Similarly, if the leader retails exclusively on-line, the change decreases by $(1/2q_1)\times(A_I - c^2_{q_2}/2)$.

more likely to intersect beyond $f_1 = 1$. The opposite effect occurs in Figure 7b. The left-hand side of Equation 9 is now higher as a function of the incumbent's output level. Keeping the right-hand side of Equation 9 fixed, we observe that f_1^* moves left (i.e. decreases) and that the entrant should again differentiate itself by retailing a sizeable proportion (if not all) of its output off-line, whenever the incumbent only retails on-line. These observations are summarized in Prescription 3(i).

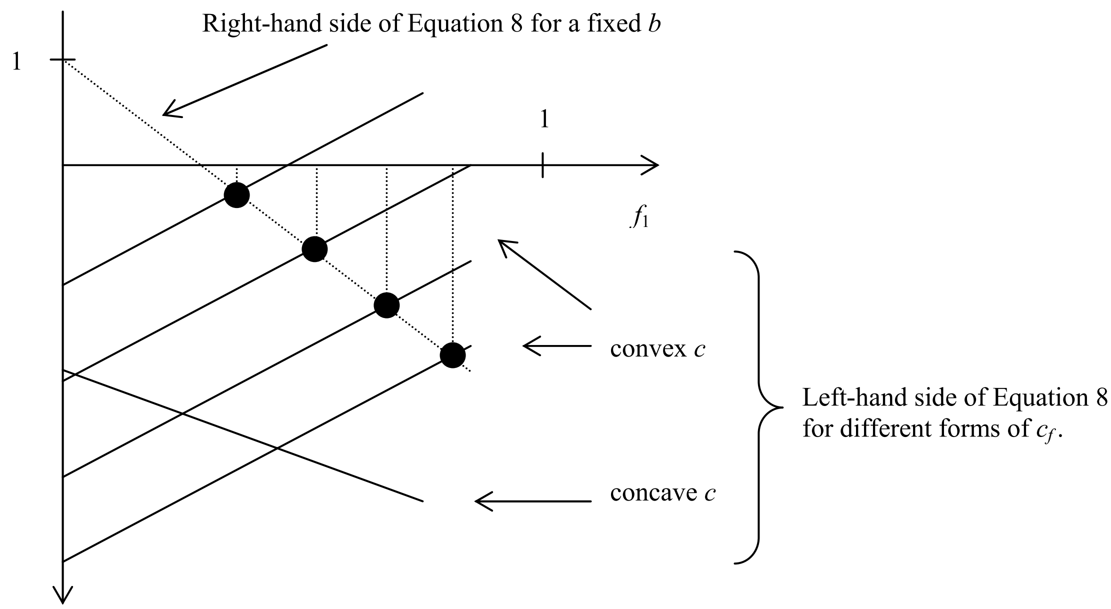

Figure 7a: Characterization of f_1^* given $f_2 = 0$

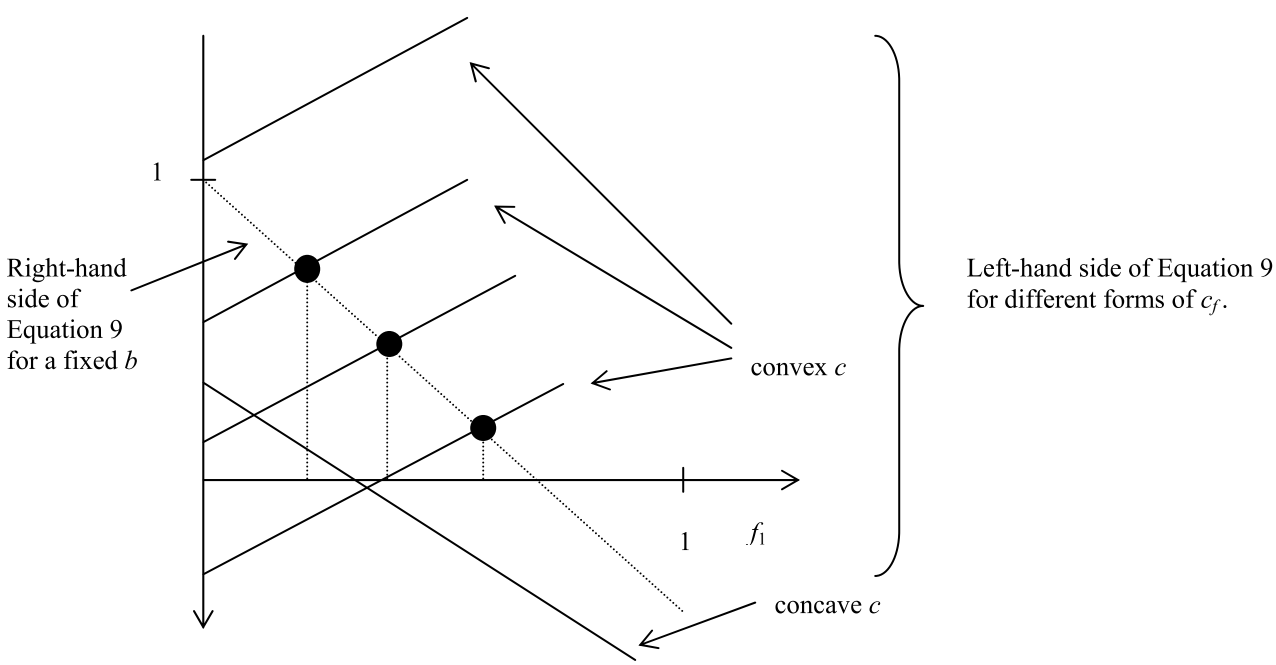

Figure 7b: Characterization of f_1^* given $f_2 = 1$

Table 3 presents the conditions under which two firms operate exclusively on one channel, $f_i^* = 0$ or 1, i=1,2, or *corner* solutions. Table 3 also reports on the conditions where one firm only retails on one channel while the other retails on two, or both firms compete on both channels, i.e. the *interior* solutions. We formally state our findings as Prescription 3(ii)-(iv).

Table 3: Conditions for corner and interior solutions ($\Delta A = A_I - A_B$)

		f_2: 0	f_2: (0,1)	f_2: 1
f_1	0	$c^i_{f_if_i} \le -2(b+1)q_i^2$, $i \in \{1,2\}$ and $(\Delta A + q_j)q_i + (1-b)q_i^2 \le c^i(1\|q_i) - c^i(0\|q_i)$ $i, j \in \{1,2\}, i \ne j$	$c^1_{f_1f_1} \le -2(b+1)q_1^2$ $c^2_{f_2f_2} > -2(b+1)q_2^2$ and $(\Delta A + q_2)q_1 + (1-b)q_1^2 - (1+b)q_1q_2f_2 \le c^1(1\|q_1) - c^1(0\|q_1)$	$c^i_{f_if_i} \le -2(b+1)q_i^2$, $i \in \{1,2\}$ and $(\Delta A + bq_2)q_1 + (1-b)q_1^2 \le c^1(1\|q_1) - c^1(0\|q_1)$ $(\Delta A + q_1)q_2 + (1-b)q_2^2 \ge c^2(1\|q_2) - c^2(0\|q_2)$
	(0,1)	$c^1_{f_1f_1} > -2(b+1)q_1^2$ $c^2_{f_2f_2} \le -2(b+1)q_2^2$ and $(\Delta A + q_1)q_2 + (1-b)q_2^2 - (1+b)q_1q_2f_1 \le c^2(1\|q_2) - c^2(0\|q_2)$	$c^i_{f_if_i} > -2(b+1)q_i^2$, $i \in \{1,2\}$	$c^1_{f_1f_1} > -2(b+1)q_1^2$ $c^2_{f_2f_2} \le -2(b+1)q_2^2$ and $(\Delta A + q_1)q_2 + (1-b)q_2^2 - (1+b)q_1q_2f_1 \ge c^2(1\|q_2) - c^2(0\|q_2)$
	1	$c^i_{f_if_i} \le -2(b+1)q_i^2$, $i \in \{1,2\}$ and $(\Delta A + bq_2)q_1 + (1-b)q_1^2 \ge c^1(1\|q_1) - c^1(0\|q_1)$ $(\Delta A + q_1)q_2 + (1-b)q_2^2 \le c^2(1\|q_2) - c^2(0\|q_2)$	$c^1_{f_1f_1} \le -2(b+1)q_1^2$ $c^2_{f_2f_2} > -2(b+1)q_2^2$ and $(\Delta A + q_2)q_1 + (1-b)q_1^2 - (1+b)q_1q_2f_2 \ge c^1(1\|q_1) - c^1(0\|q_1)$	$c^i_{f_if_i} \le -2(b+1)q_i^2$, $i \in \{1,2\}$ and $(\Delta A + q_j)q_i + (1-b)q_i^2 \ge c^i(1\|q_i) - c^i(0\|q_i)$ $i, j \in \{1,2\}, i \ne j$

PRESCRIPTION 3:

(i) If the incumbent operates exclusively on one channel and produces a large enough output volume, it will induce the new entrant to retail exclusively on the other channel.

(ii) If the product retailed by both firms is digitalizable (i.e. $c^{1.h}_{f_1} \le 0$ and $c^{2.h}_{f_2} \le 0$) then both firms always retail a positive proportion of their output on-line.

(iii) Each firm must use dissimilar technologies for one firm to retail on two channels when its competitor operates exclusively on one.

(iv) Rarely should both firms choose to distribute on both retail channels.

Result (i) is a consequence of our decoupled markets, as one market gets saturated the incumbent capitalizes on a larger profit opportunity from the other market. Result (ii) follows from the fact that the only possible corner solution for a digitalizable product is for both firms to exclusively retail on-line. This result is congruent with intuition. The said conditions pin the incremental revenues from moving on-line (always positive since the on-line market is, by assumption, larger than the off-line) against the corresponding cost increases, which are negative by definition. In (iii) such outcomes can occur whenever each firm uses dissimilar technologies since their cost function's concavity is bounded below (firm 1) and above (firm 2). In (iv) the conditions of Table 3 only impose a lower bound on the concavity (in f_i) of each firm's cost function. This entails quick savings from moving on-line that decrease as the on-line position increases. Only firms that exhibit this somewhat restrictive (and unlikely) cost structure would benefit from competing on both channels of distribution.

We also investigated a simultaneous-game scenario where each firm's cost structure was linear in output given the on-line strategy and where these firms competed on a single market. Findings were consistent with that of the above models. The equilibrium for digitalizable products called for exclusive on-line retailing for both firms and, for solid products, exclusive off-line retailing when their cost functions increase in the on-line proportion. Technical details and explanations for these equilibria are offered in the appendix.

7. Conclusion

In this paper we showed that the decision to retail on-line involves more than the lure of additional revenues from selling to an expanded market that the Internet provides. The supply chain technological attributes (embodied in the cost function) along with the product's key characteristics (digitalizable or solid) should be the drivers behind how much product a new entrant should sell on-line or off-line. We demonstrated that there are indeed profitability (monopoly models) or competitive (duopoly models) advantages in a firm's strategic choice of distribution channels. For a monopolistic new firm, digitalizable product should be solely retailed on-line, as seen with financial institutions, transportation & accommodation suppliers, software and data retailers that have successfully shifted the retailing of their digitalizable goods to an on-line channel (even when operating in competitive environments). Solid products, on the other hand, should be retailed on one or two channels based on the behavior of the monopolist's inventory & distribution costs and retailing costs, as retailing a solid good on-line may cause an increase in supply chain costs that on-line revenues will not outweigh. When the monopolistic new firm faces an economy with two markets (off- and on-line), it may benefit from retailing a digitalizable product on both channels whenever its cost function, despite being decreasing, does not capture enough savings from moving more output on-line.

For an entrant facing an already populated market, we analyzed a leader-follower scenario where, given an incumbent monopolist operating exclusively on one channel, a potential entrant to the industry must decide how much of its output should be retailed on- and off-line. We prescribe that whenever the incumbent operates exclusively on one channel and produces a large enough output volume the entrant should retail exclusively on the other channel, therefore revealing a new competitive strategy in the proportion of output one firm sells on-line where as that proportion increases for the incumbent, it will decrease for the entrant, and vice versa. Also, if the product retailed by both firms is digitalizable then both firms should always retail a positive proportion of their output on-line. Moreover, each firm must use dissimilar technologies (cost structures) for one firm to retail on two channels when its competitor operates exclusively on one, and rarely should both firms choose to distribute on both retail channels.

There are many ways in which a new entrant can harness the Internet to benefit its business. In recent years, we have observed the evolution of on-line retailing, from new firms offering an exclusive on-line presence to traditional bricks-and-mortar firms opening on-line stores. Successful on-line retailers today are those that have been able to keep their operating costs low with sophisticated inventory management systems, efficient contracts with third party shippers, and strategic alliances with existing bricks-and-mortar outlets. Prescribing alternative strategies such as restricting Internet presence by only keeping an on-line listing or advertising products (but only selling through toll-free numbers or bricks-and-mortar outlets) may fall outside our model scope. We have assumed the choice set of the firm to be restricted to the proportion of output sold on-line, and that all the firm's performance and technology could be embodied in the three-tiered cost function (Equation 1).

There are a number of useful directions for extending this research to capture more of the complexity in retailing decisions. One is to add marketing costs to the model, which would increase the precision of our results. However, our preliminary analysis shows that such costs render the derivation of useful results from our current models quite tedious. This suggests that numerical or empirical analyses would be better suited to capture the insightful trade-offs between marketing and supply chain costs, when a firm has the option of operating on more than one channel. There is much still to be learned on the virtual/brick-and-mortar decision and this decision is central to a firm's performance. Still, this new research coupled with our findings will help new entrants make more informed decisions about which channels to utilize.

REFERENCES

Anupindi, R., Y. Bassok and E. Zemel, "A general framework for the study of decentralized distribution systems," *Manufacturing & Service Operations Management*, Vol. 3, No. 4:349-368, Fall 2001.

Bakos, Y., "The emerging landscape for retail E-commerce," *Journal of Economic Perspectives*, Vol. 15, No. 1:69-80, 2001.

Bakos, Y. and E. Brynjolfsson, "Bundling and competition on the Internet," *Marketing Science* , Vol. 19, No. 1:63-82, Winter 2000.

Balasubramanian, S., "Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers," *Marketing Science*, Vol. 17, No. 3:181-195, Fall 1998.

Chopra, S. and J.A. Van Mieghem, "Which e-business is right for your supply chain?," *Supply Chain Management Review*, July-August 2000.

Devinney, T.M., A. Latukefu and D. F. Midgley, "E-business: revolution, evolution, or hype?," *California Management Review*, Vol. 44, No. 1:57, 2001.

Dewan, R., B. Jing and A. Seidmann, "Adoption of Internet-based product customization and pricing strategies," *Journal of Management Information System*, Vol. 17, No. 2:9-28, 2000.

Fudenberg, D. and J. Tirole, *Game Theory*. The MIT Press, Cambridge, MA, 1993.

Gulati, R., and J. Garino, "Get the right mix of bricks and clicks," *Harvard Business Review*, Vol. 78:107-114, May-June 2000.

Hendershott, T. and J. Zhang, "A model of direct and intermediated sales," Working paper, University of California at Berkeley and University of Rochester, 2001.

Hitt, L.M. and F.X. Frei, "Do better customers utilize electronic distribution channels? The case of PC banking," *Management Science*, Vol. 48, No. 6:732-748, June 2002.

Hoffman, D.L., "The revolution will not be televised: Introduction to the special issue on Marketing Science and the Internet," *Marketing Science,* Vol. 19, No. 1:1-3, Winter 2000.

Jaffe, R., "Multi-channel vs pureplay philosophies appear night and day," PaineWebber Research Note, Vol. 24:1, August 2000.

Kaufman-Scarborough, C. and J. D. Lindquist, "E-shopping in a multiple channel environment," *Journal of Consumer Marketing*, Vol. 19, No. 4:333-350, Fall 2002.

Keeney, R.L., "The value of Internet commerce to the customer," *Management Science*, Vol. 45, No. 4:533-542, April 1999.

Kellee, H., "Vendors selling direct online," *Sporting Goods Business* , Vol. 34, No. 11:18, August 2001.

Leamer, E.E. and M. Storper, "The economic geography of the Internet age," *Journal of International Business Studies*, Vol. 32, No. 4: 641-665, 2001.

Levin, A.M., I.P. Levin I.P. and C.E. Heath, "Product category dependent consumer preferences for online and offline shopping features and their influence on multi-channel retail alliances," *Journal of Electronic Commerce Research*, Vol. 4, No. 3:85-93, August 2003.

Lucking-Reiley, D. and D. F. Spulber, "Business-to-business electronic commerce," *Journal of Economic Perspectives*, Vol. 15, No. 1:55-68, Winter 2001.

Messinger, P.R. and C. Narasimhan, "A model of retail formats based on consumers' economizing on shopping time," *Marketing Science,* Vol. 16, No. 1:11-23, March 1997.

Salop. S., "Monopolistic competition with outside goods," *Bell Journal of Economics,* Vol. 10:141-156, 1979.

Schoenbachler, D.D. and G. L. Gordon, "Multi-channel shopping: understanding what drives channel choice," *Journal of Consumer Marketing*, Vo. 19, No.1:42-53, 2002.

The Economist, "Shopping around the web. A survey of e-commerce," February 26, 2001.

Tsay, A.A. and N. Agrawal, "Manufacturer and reseller perspectives on channel conflict and coordination in multiple-channel distribution," Working paper, Santa Clara University, 2001.

Vishwanath, V. and G. Mulvin, "Multi-Channels: The Real Winners in the B2C Internet Wars," *Business Strategy Review*, Vol. 12, No. 1:25-33, 2001.

APPENDIX

Technical details for section 5.1. One market (solving for firm output *Q*)

The monopolist now solves for total output Q^{M}, and must thus

$$\operatorname*{Max}_{Q} \, (A-Q)\times Q \; - c(Q, f^{*}), \tag{A1}$$

with $f^{*} = f(Q)$ elicited in Prescription 1. The profit maximizing quantity Q^{M} is the solution to

$$Q = \frac{1}{2}\times\left\{ A - \frac{\partial c\big(Q, f^{*}(Q)\big)}{\partial Q} - \frac{\partial c\big(Q, f^{*}(Q)\big)}{\partial f}\times\frac{df^{*}(Q)}{dQ}\right\}. \tag{A2}$$

The general nature of Equation A1 hinders the derivation of an intuitive closed form solution. The profit maximizing quantity Q^{M} that induces $f^{*} = 1$ ($f^{*} = 0$) is the solution to $Q = \frac{A - c_{Q}(Q,1)}{2}$ (respectively, $Q = \frac{A - c_{Q}(Q,0)}{2}$). From basic micro-economics, we observe that these two quantities are exactly the profit maximizing levels for a monopolist, given a linear demand curve and a general marginal cost function.

Technical details for section 5.2. Two markets (solving for firm output *Q*)

The monopolist solves for total output Q^{M}, and must thus

$$\operatorname*{Max}_{Q}\big(A_{I} - bf^{*}Q\big)\times f^{*}Q \;+\; \big(A_{B} - (1-f^{*})Q\big)\times(1-f^{*})Q \;-\; c(Q, f^{*}), \tag{A3}$$

with $f^{*} = f(Q)$ elicited in Equation 4. Here again, the general nature of Equation A3 hinders the derivation of an intuitive closed form solution. We nonetheless note that the profit maximizing quantity Q^{M} that induces $f^{*} = 1$ is the solution to $Q = \frac{A_{I} - c_{Q}(Q,1)}{2b}$ and the analogous quantity that induces $f^{*} = 0$ is the solution to $Q = \frac{A_{B} - c_{Q}(Q,0)}{2}$. These two quantities correspond to the standard profit maximizing output level for a monopolist, given the appropriate demand curve and marginal cost function. We observe that, for digitalizable products with a more elastic on-line demand ($b \leq 1$), the former quantity dominates the latter one. Aside from standard economic intuition (optimal output level depends on both market structures and on the firm's cost function), deriving a general form of Q^{M} reveals little additional insight over the results already stated.

Technical details for section 6 Duopoly Model

The first and second order derivatives of Equation 7 are, respectively,

$$\frac{\partial \Pi_{1}}{\partial f_{1}} = -2(1+b)q_{1}^{2} f_{1} + (A_{I} - A_{B})q_{1} + 2q_{1}^{2} + q_{1}q_{2}[1-(1+b)f_{2}] - c_{f_{1}}^{1} \tag{A4}$$

$$\text{and} \quad \frac{\partial^{2}\Pi_{1}}{\partial f_{1}^{2}} = -2(1+b)q_{1}^{2} - c_{f_{1}f_{1}}^{1}. \tag{A5}$$

Assuming that c^{1} is convex in f_{1}, then setting Equation A4 to zero is sufficient to maximize Equation 7. This yields

$$\frac{c_{f_{1}}^{1} - (A_{I} - A_{B})q_{1} + q_{2}[1-(1+b)f_{2}]q_{1}}{2q_{1}^{2}} = 1-(1+b)f_{1}. \tag{A6}$$

Equation A6 together with the fact that firm 2 is the leader and a monopolist operating only on one channel (and thus we can fix $f_{2} = 0$ or 1 and $q_{2} = \frac{A_{B} - c_{q_{2}}^{2}}{2}$ or $q_{2} = \frac{A_{I} - c_{q_{2}}^{2}}{2b}$, respectively) lead to Equations 8 and 9 and hence Prescription 3(i).

Conditions for interior and corner solutions. Consider Equations 7, A4 and A5. We note that $f_{1} = 0$ maximizes $\Pi_{1}\big(f_{1} \mid q_{1}; f_{2} = 0, q_{2}\big)$ whenever, from Equation A5, $c_{f_{1}f_{1}}^{1} \leq -2(b+1)q_{1}^{2}$, and whenever

$\Pi_1(f_1 = 0 \mid q_1; f_2 = 0, q_2) \geq \Pi_1(f_1 = 1 \mid q_1; f_2 = 0, q_2)$ or, equivalently, $(\Delta A + q_2)q_1 + (1-b)q_1^2 \leq$ $\leq c^1(1 \mid q_1) - c^1(0 \mid q_1)$ where $\Delta A = A_I - A_B$. If $f_1 = 0$ we see that setting $f_2 = 0$ maximizes $\Pi_2(f_2 \mid q_2; f_1 = 0, q_1)$ if $c^2_{f_2 f_2} \leq -2(b+1)q_2^2$, and if $\Pi_2(f_2 = 0 \mid q_2; f_1 = 0, q_1) \geq$ $\Pi_2(f_2 = 1 \mid q_2; f_1 = 0, q_1)$ or, equivalently $(\Delta A + q_1)q_2 + (1-b)q_2^2 \leq c^2(1 \mid q_2) - c^2(0 \mid q_2)$. This result is reported in Table 3. In addition, Table 3 presents all conditions for the other (leader-follower) corner and interior solutions. Additional technical details for deriving these conditions are omitted as they parallel the above development.

Prescription 3(ii) is derived as follows. By definition, we know that $c^{i,h}(1 \mid q_i) - c^{i,h}(0 \mid q_i) \leq 0$, $i = 1,2$, for solid products. Therefore, the conditions for $f_1 = f_2 = 0$, $f_1 = 0$ and $f_2 = 1$, or $f_1 = 1$ and $f_2 = 0$ of Table 3 can never be achieved. Moreover, the only possible corner solution for a digitalizable product is $f_1^* = f_2^* = 1$. For Prescription 3(iii), consider solutions where firm 2 sets $f_2 = 0$ or $f_2 = 1$ and firm 1 prefers a strategy in (0,1) (we can conduct a similar analysis for $f_1 = 0$ or $f_1 = 1$ and firm 2 prefers a strategy in (0,1)). An interior solution obtains whenever firm 1's cost function is, in f_1, convex (or not too concave), i.e. $c^1_{f_1 f_1} > -2(b+1)q_1^2$ whereas firm 2's is strictly concave, i.e. $c^2_{f_2 f_2} \leq -2(b+1)q_2^2$. For such outcomes to occur, each firm must use dissimilar technologies since their cost function's concavity is bounded below (firm 1) and above (firm 2). Finally, for Prescription 3(iv), we focus on cases where both firms choose to distribute on two retail channels, $f_i \in (0,1)$, $i = 1,2$. Here, the conditions only impose a lower bound on the concavity (in f_i) of each firm's cost function.

Nash equilibria for two firms with linear cost functions

In this somewhat restrictive scenario, firm i's cost function is linear and given by $c^i(q_i \mid f_i) = q_i \times \delta^i(f_i)$ where we assume δ^i, $i = 1,2$ to be non-negative and differentiable on the interval [0,1]. Observe from Table 1 that if firm i retails a digitalizable product, then its cost is increasing in its on-line strategy ($d\delta^i / df_i > 0$) but that the reverse is not necessarily true, i.e. retailing a solid product does not mean that the firm's cost is decreasing in its on-line strategy ($d\delta^i / df_i < 0$). In addition, we assume that both firms compete on a single market that has linear demand given by A-Q. Furthermore, we make a structural assumption for feasibility where $A + \delta^j(f_j) - 2\delta^i(f_i) \geq 0$ for any f_i, f_j in [0,1], $i \neq j$, $i, j = 1,2$. This assumption ensures that the equilibrium in output is non-negative for each firm. Firm i maximizes its profits given by $\Pi_i = (A - q_i - q_j) \times q_i - c^i(q_i, f_i)$, $i \neq j$, $i,j = 1,2$.

PRESCRIPTION A1:

(i) Digitalizable products should never be exclusively retailed off-line, but solid products could whenever firms' costs are increasing in their on-line strategy f_i.

(ii) Solid products should never be exclusively retailed on-line unless firms' costs are decreasing in their on-line strategy f_i, but a Nash equilibrium exists where digitalizable products should be exclusively retailed on-line.

(iii) A separation strategy where one firm exclusively retail on-line whereas the other firm has an exclusive presence off-line is never optimal for digitalizable products.

(iv) It is never optimal for one firm to retail a digitalizable product on two channels when its competitor operates exclusively on one.

PROOF: We note that $\partial^2\Pi_i / \partial q_i^2$ and thus the first-order condition is sufficient for a maximum. Moreover, since $\left|\partial^2\Pi_i / \partial q_i^2\right| = 2 > 1 = \left|\partial^2\Pi_i / \partial q_i \partial q_j\right|$ there exists a crossing between the firms' reaction function (Fudenberg and Tirole, 1993) and hence an equilibrium in output where $q_i^*(f_i, f_j) = \frac{1}{3}[A + \delta^j(f_j) - 2\delta^i(f_i)]$ and $\Pi_i^*(f_i, f_j) = \frac{1}{3}[A + \delta^j(f_j) - 2\delta^i(f_i)]^2$ for $i \neq j$, $i, j = 1,2$.

We now detail conditions under which corner and interior solutions are obtained. First, recall that $f_1^* = f_2^* = 0$ means that both firms operate exclusively off-line. To achieve this, the following conditions must hold

$\Pi_1^*(f_1,0)=\frac{1}{3}[A+\delta^2(0)-2\delta^1(f_1)]^2 \leq \frac{1}{3}[A+\delta^2(0)-2\delta^1(0)]^2=\Pi_1^*(0,0) \quad \forall f_1$

and $\Pi_2^*(0,f_2)=\frac{1}{3}[A+\delta^1(0)-2\delta^2(f_2)]^2 \leq \frac{1}{3}[A+\delta^1(0)-2\delta^2(0)]^2=\Pi_2^*(0,0) \quad \forall f_2$,

or $\arg\min\alpha^1(f_1)=0$ and $\arg\min\alpha^2(f_2)=0$. From these, we readily conclude that digitalizable products should never be exclusively retailed off-line (from Table 1 it follows that $d\delta^i / df_i < 0$), but solid products could whenever $d\delta^i / df_i > 0$ (Prescription A1i). Analogously, for both firms to retail exclusively on-line ($f_1^* = f_2^* = 1$) one must have $\arg\min\delta^1(f_1)=1$ and $\arg\min\delta^2(f_2)=1$. Hence, a Nash equilibrium exists where digitalizable products should be exclusively retailed on-line, but solid products should not unless $d\delta^i / df_i < 0$ (Prescription A1ii). For $f_1^* = 0, f_2^* = 1$ and $f_1^* = 1, f_2^* = 0$ one must have $\arg\min\delta^1(f_1)=0$, $\arg\min\delta^2(f_2)=1$ and $\arg\min\delta^1(f_1)=1$, $\arg\min\delta^2(f_2)=0$, respectively. Therefore, for digitalizable products we do not observe the separation effect as we did in the previous section (Prescription A1iii).

Finally, for digitalizable products a solution form such as $f_1^* = 0, f_2^* \in (0,1)$ exists as long as (a) $\arg\min\delta^1(f_1)=0$, (b) $[A+\delta^1(0)-2\delta^2(f_2)]\times[d\delta^2 / df_2]=0$ (1st order condition for interior solution), and, given that (b) holds, (c) $\left(d\delta^2 / df_2\right)^2 < 0$ (2nd order condition for interior solution) which is impossible. Similar conditions must hold for interior solutions $f_1^* \in (0,1)$ & $f_2^* = 0$, $f_1^* = 1$ & $f_2^* \in (0,1)$, $f_1^* \in (0,1)$ & $f_2^* = 1$, and $f_1^* \in (0,1)$ & $f_2^* \in (0,1)$. Thus, it is never optimal in this model for one firm to retail a digitalizable product on two channels when its competitor operates exclusively on one (Prescription A1iv). ■

(i) and (ii) follow from intuition. Indeed, profit maximization imposes on both firms that they flow an optimal volume on each channel. We obtain those results since the on-line channel is more expedient for digitalizable product, as both the inventory & distribution and the retailing costs (and therefore total cost) decrease with the on-line proportion f_i. The off-line channel is more expedient for solid products whenever the inventory & distribution cost is over and above that of retailing, thus making total cost increasing in the on-line proportion. However, (iii) is somewhat counter-intuitive since we have demonstrated before that a separation strategy can be called for even for digitalizable products. Still, under our single market assumption, there is no guarantee that exclusively using one channel when the other is occupied is warranted since the other channel does not provide access to additional customers. Finally, (iv) follows from the model's premises, namely the linear cost function which makes sub-optimal solutions where a firm retails a digitalizable product on both channels.