ANALYSIS OF ELECTRONIC MICRO-PAYMENT MARKET

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ABSTRACT

Despite the potential of micro-payment systems very few systems have been successful. Little is known about the reasons behind the successful few and the failures of the majority. Micro-payment markets exhibit two-sided network effects and the underlying dynamics of these markets are not very well understood. Based on a stylized model of a two-sided market, we find that a 'survival mass' of merchants and consumers is required for a micro-payment system to exist and a 'critical mass' for the acceptance levels to take off and remain stable. We also find the non-intuitive result that lowering the consumer-side adoption cost will actually reduce the chances for the micro-payment market to develop. Thus, subsidization alone cannot create a micro-payment market. Anecdotal evidence supports this finding. When subsidization is needed, the consumer side will normally be subsidized. The two-sided market structure makes comparative analysis complex and non-trivial, rendering the implementation of micro-payment systems very difficult as indicated by the mixed results of a number of initiatives worldwide.

Keywords: Network Effects, Two-sided Market, Micro-payment Systems, Smart Card Technology, Electronic Cash, Game Theory

1. Introduction

The idea of having a cashless world has long been around. The costs of handling cash are high compared to that of electronic money. Printing, distributing and controlling cash are estimated to cost a developed economy 0.75% of annual GDP and an emerging economy 1% to 2% (Cobb, 2003). Social savings of using electronic micro-payment means over cash are substantial.

Given the huge potential savings electronic micro-payment can bring about, there is scope to increase profits. Electronic micro-payment is also important for a wide range of electronic and mobile commerce (Harris and Spence 2002, Papaefstathiou and Manifavas 2004), which further enhances the incentives for firms to enter this market (Baddeley 2004). Consequently, major credit card operators and financial institutions had been trying to capitalize on this business throughout the 90s. Initiatives like Mondex and Visa Cash (Westland, 1998; Westland et. al., 1997) have achieved little success. In an early pilot test (Van Hove, 2000), the acceptance levels of both Mondex and Visa Cash were disappointing.

An exception is the Octopus card in Hong Kong which ranks top worldwide in terms of circulation (Yoon, 2001). The Octopus card was originally a fare-payment smart card for the Hong Kong passenger transportation system. A joint venture firm called Creative Star Limited was formed by the five major public transportation operators to develop the system. It was introduced to the public in 1997, targeting a public transportation market with 10 million passenger journeys per day and total daily transactions valuing over 2.5 million dollars (Poon and Chau, 2001). A critical mass was quickly gained and the Octopus card system is now growing to support non-transportation micro-payment transactions too. With over 7 million cards issued, it is now the closest thing to an electronic-cash system anywhere in the world (Yoon, 2001).

The Octopus card has attracted significant attention (CPSS Survey, 2001) and its success is frequently contrasted with the failure of Visa Cash and Mondex which were launched contemporaneously in Hong Kong. The overwhelming success of the Octopus card is surprising. It does not have a gigantic client base compared with Visa Cash and Mondex, which are supported by Visa Card and Master Card respectively. Neither is the Octopus card more secured (Hong Kong Economic Times, 2000).

We believe the mixed outcomes of different micro-payment systems are attributed to the very basic market structure and the complex interdependent relationship between merchants and consumers that are not intuitive to a system provider. In particular, a micro-payment market can be conceptualized as a two-sided market involving consumers and merchants subject to positive network effects on both sides. This unique market structure has two important implications. First, the demand for a micro-payment system must come jointly from consumers and merchants. Second, system providers face the well-known chicken and egg dilemma: consumers want merchants on board and vice versa. The underlying dynamics of these two-sided markets are not very well understood and this may lead to the mixed results of various micro-payment initiatives.

To investigate the implications of such a two-sided market structure, we study factors affecting the existence of a market for micro-payment systems, how the consumers and merchants choose their acceptance levels, and how a profit maximizing system provider sets the prices. Network effects that arise from the usage of such micro-payment systems play an important role. Normally, network effects derive directly from the number of users in the network. Telecommunication networks are a common example. Network effects of electronic payment systems are different - the value of the system to consumers depends on the number of merchants adopting the system and vice versa. These basic relationships and their effects on strategic behavior of each stakeholder of a micro-payment system can best be studied in the context of a stylized analytical model. We are interested in the following research questions: What determines the consumer and merchant acceptance levels? What are the factors affecting the existence of a market for a micro-payment system? What are the pricing guidelines for system providers?

Our findings have important managerial implications. We find that a “survival mass” of merchants and consumers is required for the market to exist and a “critical mass” for the acceptance levels to take off and remain stable. There is also a lower bound for the consumer and merchant demands. The conditions for the existence of the market are derived. We find the non-intuitive result that lowering the consumer-side adoption cost will actually reduce the chances for the micro-payment market to develop. We also find that subsidization alone cannot create a micro-payment market. An early pilot test on Visa Cash and Mondex (Van Hove, 2000) supports this result. The two-sided market structure makes comparative statics complex and non-trivial. The more benefits consumers can get from the system, the lower the optimal merchant prices and consumer acceptance levels will be. The optimal merchant acceptance levels are unaffected by decreases in costs for consumers to adopt the system. Similarly, the more benefits merchants can get from the system, the lower the optimal consumer prices and merchant acceptance levels will be. The optimal consumer acceptance levels are unaffected by decreases in costs for merchants to adopt the system. Also we find that when subsidization is needed, the consumer side will be subsidized. The merchant side will receive subsidization only under very rare conditions. To achieve full acceptance levels, the system provider needs to subsidize both consumers and merchants.

The rest of the paper is organized as follows: Section 2 briefly reviews previous studies related to electronic payment systems, network effects and multiproduct pricing. Section 3 describes the model used. Section 4 analyses the interaction between consumers and merchants. Section 5 lays out the analysis of the pricing decisions in a monopoly market. Section 6 concludes and discusses the results, and identifies future research directions.

2. Literature Review

When the value of a product depends on the number of users, the product exhibits network effects. When the value increases with the number of users, there are positive network effects. For example, a telephone network is more valuable when there are more users with whom one can communicate. Products can also exhibit negative network effects. For example, when there are too many cars, i.e. users, on the road, undesirable traffic jams may occur.

However, in electronic micro-payment systems, network effects are more complex. There is a joint demand requirement and a chicken-and-egg dilemma. Merchants will not adopt the system unless there are enough consumers who use the system. At the same time, consumers will not consider the system until there are enough adopters on the merchant side. Figure 1 depicts this market structure.

Thus, the value of a micro-payment system to a consumer increases as more merchants adopt the system. Similarly, the value of the system to a merchant increases as more consumers are willing to transact with the merchant using the new electronic means. This dilemma makes the market dynamics complicated and there are no straightforward guidelines for system providers to set their prices. In previous research, these types of interdependent network effects are called two-sided network effects (Yoo et. al., 2002). We adopt this term. For network effects in traditional networks such as phone networks, email networks, etc., we refer to them as one-sided network effects.

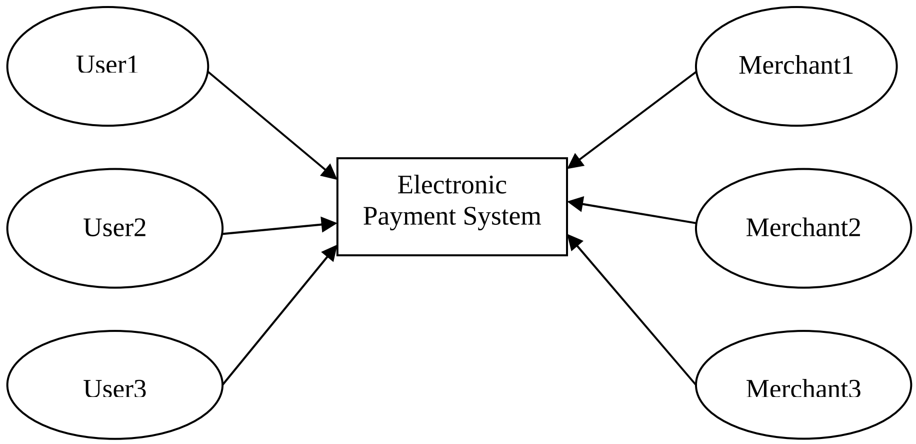

Figure 1: Interdependence of consumers and merchants

Network effects from one-sided markets form a broad stream of economic literature. Economides (1996) provides an excellent survey. Research has shown that one-sided network effects have significant impacts on firm strategies and consumer behaviors. For example, studies in the spreadsheet market demonstrate how network effects cause higher prices and set common standards (Brynjolfsson and Kemerer, 1996; Gandal, 1994). Furthermore, a number of studies (Au and Kauffman, 2001; Tam and Hui, 2001) reported the importance of network effects in technology adoption decisions. Gupta, Jukic, Stahl and Whinston (2000) investigated the effects of negative network effects on internet traffic pricing schemes.

While this research stream advances our understanding of one-sided network markets, there is relatively little research investigating two-sided network effects. A few that had been studied are Electronic Data Interchange (EDI) networks, electronic payment systems, B2B marketplace, and the general aspects of two-sided markets. The two sides involved in EDI networks are buyers and sellers. In studies of EDI networks, the two-sided nature of the market had not been the focus, and network effects were treated the same as in one-sided markets. Wang and Seidmann (1995) showed the presence of both positive and negative network effects. Riggins, Kriebel, and Mukhopadhyay (1994) investigated how a buyer can attract suppliers to its EDI network while Barua and Lee (1997) studied how subsidizing suppliers can increase the adoption of an EDI system.

Most studies on adoption and usage of a new electronic payment technology have been based on the Technology Acceptance Model (Davis 1989) and innovation diffusion theory (Rogers 1983; 1995), without explicit consideration of the two-sided market structure. Plouffe et al. (2001a) studied adoption decisions of a smart card based micro-payment system from the merchant side, in the context of a market trial. In a similar context, Plouffe et al. (2001b) surveyed the perceptions from adopting and non-adopting consumers and merchants. Both studies found that the relative advantages of a micro-payment technology and its compatibility with existing consumer experiences were two important variables across different adoption groups. Shy and Tarkka (2002) and Van Hove (2000) are two studies using a different theoretical perspective. Shy and Tarkka (2002) explained the co-existence of micro-payment and other payment means theoretically by transaction costs. Van Hove (2000) illustrated that network effects and communication channels, such as advertising and word-of-mouth, are the two most important factors governing the adoption decision for an electronic payment system of consumers and merchants. System providers can use communication channels to disseminate information in their favor so that consumers and merchants are more willing to adopt by having a better understanding of the benefits of using the electronic payment system.

Two-sided network effects have been studied in general, and in the context of B2B marketplaces. Yoo et al. (2002) analyzed a monopolistic B2B marketplace and found that the optimal pricing strategies were simultaneously dependent on the switching costs faced by participants, the strength of the two-sided network effects, and the nature of industry served. Rochet and Tirole (2001) investigated two general aspects of two-sided markets: the price allocation and welfare implications. Their work linked network economics with multiproduct pricing by analyzing the price allocation question in the presence of two-sided network effects.

While two-sided network effects have been studied in the context of B2B marketplaces, micro-payment systems differ from B2B marketplaces in three important ways. First, consumers and merchants are two clearly separate entities while a participant of a B2B marketplace could act as both a buyer and a seller. Second, we have no negative network effects for the market of electronic micro-payment systems. B2B marketplaces could exhibit significant negative network effects due to competition for business among participants. Finally, when there is no acceptance on any side of the market, an electronic payment system has no value while a B2B marketplace can still generate value by providing information. Rochet and Tirole (2001) proposed a generic model which provides a valuable starting point in analyzing two-sided markets. However, issues important to our context, such as the asymmetrical nature of merchants and consumers, are not considered. For example, adoption of a micro payment system frequently benefits merchants more than consumers, but it costs more for merchants than consumers to install and use the system. The determinants of acceptance levels in such markets are also not very well understood.

We develop a model to examine the non-cooperative outcome in a game setting consisting of three players: a monopoly micro-payment system provider, merchants and consumers. Each player aims at maximizing his own payoff. With two-sided network effects in effect, the strategic interaction among the three players is our focus in this analysis. Details of the model are presented in the next section.

3. Model Specifications

Consider a market with three parties: a micro-payment system provider, a number of merchants, and a number of consumers. Consumers and merchants trade with each other and the trades are settled with transactions of small amounts i.e. micro-payments. Unlike large transactions that are typically settled by credit card or wire-transfer, micro-payment transactions are typically anonymous. Because of the low monetary value of each transaction, security, non-repudiation, and risk are not critical factors as compared with convenience and ubiquity in driving the adoption of a micro-payment system. Hence, our model will focus on the cost and benefits of adoption for both merchants and consumers. Given the nature of micro-payment, the model focuses on the number of transactions rather than the amount of each transaction. The Octopus card system is an example of a micro-payment based market – the joint venture firm is the system provider, the different public transportation operators are the merchants and the general public are the consumers. A transaction takes place when a consumer pays a public transportation operator with the Octopus card.

A two-stage game is used to model and study the strategic behaviour of each player. Each player knows others' payoffs. In stage one, the system provider offers a pair of prices P_m and P_u to merchants and consumers respectively to use the system. In stage two, both consumers and merchants observe the offered prices and then move (decide whether to adopt) simultaneously. The resulting proportion of adopted consumers and merchants are D_u and D_m respectively. Since D_u and D_m are proportions, we have $0 \le D_u, D_m \le 1$.

3.1 Consumer preference

Consumers are heterogeneous in the number of transactions made with the merchants. A consumer's type corresponds to the number of transactions he/she makes. Two consumers are of the same type if they make the same number of transactions. A consumer i does a certain number of transactions $\theta_i q$. The type of a consumer i is denoted by θ_i and we assume that θ_i is uniformly distributed over [0,1]. The transaction frequency of a particular consumer market is captured by q.

There are several benefits an adopted consumer can get each time he/she uses the micro-payment system rather than cash. For example, the consumer can decrease the time needed for the payment process and the process of payment is also simplified in most cases. For instance, when paying for transportation fares in Hong Kong, an Octopus card consumer can decrease the time for payment and avoid the troubles in handling change. Other factors remain the same, a higher q will entail a consumer to adopt the system because of a larger increase in convenience (i.e. benefit) as compared with a lower q.

Time and convenience constitutes a major benefit of adopting and using an electronic payment system. This comes from the contact-less nature of the card in most cases. Technology plays a dominant role in realizing the system benefits. All benefits per transaction are summarized and represented by b^U. This benefit is a per-period benefit.

Previous research suggested that the nature of transactions is important to consumer preference (Van Hove, 2000). Some types of transactions have a higher value to consumers. Unattended point-of-sales applications are one example. These are uses for which cash is truly inconvenient (Clemons et. al., 1996; Weaver, 1998). Under this model, these considerations can be easily handled by using different values for b^U.

By observing prices P_m and P_u, consumers form an expectation on merchant acceptance levels. At equilibrium, this expectation equals the resulting merchant acceptance D_m (Katz and Shapiro, 1985). The more merchants adopting the system, the higher the chance a consumer can settle a micro-payment transaction using the system. Consumers cannot use the system if no merchant adopts it. In short, consumers benefit from having more merchants on board i.e. consumers prefer a higher D_m. Consequently, a consumer who makes $\theta_i q$ transactions gets a gross benefit $D_m \theta_i q b^U$.

The gross benefit $D_m \theta_i q b^U$ is dependent on the network effect of the merchant market to a consumer i. Therefore, the intensity of the network effect is determined by the type of consumer. In other words, the consumer's frequency of using the micro-payment system defines his/her intensity of the network effect.

If a consumer decides to adopt the electronic payment system, he/she needs to learn how to use the micro-payment system. We denote this learning cost by ϕ^U, which is assumed the same for all consumers[1]. This learning cost is a per-period cost. System design features will be one important determinant for the learning cost. For example, using a contact-less smart card is considered to be simpler and easier to learn than using a magnetic card.

Summarizing all the above, the net surplus of a particular consumer *i* is:

$$U_i = D_m \theta_i q b^U - P_u - \phi^U$$

if he/she decides to adopt the electronic payment system. Otherwise, the consumer gets the reservation utility. Without loss of generality, we assume a zero reservation utility. Therefore, we have $U_i = 0$ if the consumer decides not to adopt the payment system. We further denote the overall consumer-side system benefits qb^U, which is partly due to the transaction frequency q, and partly due to the technology b^U, by B^U. Obviously, a consumer will adopt the electronic payment system if and only if

$$U_i = D_m \theta_i B^U - P_u - \phi^U \geq 0$$

The table below summarized the consumer parameters in the model, and its linkage to real world examples:

Table 1: Consumer side model parameters

Parameters	**Description**	**Examples from the Octopus case**
q	The transaction frequency of the consumer side of a particular market	Octopus, when first launched, was targeting a public transportation market with 10 million passenger journeys per day.
b^U	Consumer benefits per transaction	When paying for transportation fares in Hong Kong, an Octopus card consumer could decrease the time for payment and avoid the trouble of handling change.
ϕ^U	The cost of learning to use the system	System design features will be one important determinant for the learning cost. Octopus is a contact-less smart card, which is considered to be simpler and easier to learn than using a magnetic card.

3.2 Merchant preference

Similarly, merchants are heterogeneous in the number of transactions they make with consumers. The number of transactions each merchant makes defines her type. A merchant j makes $\vartheta_j n$ transactions. Hence the type of a merchant j is represented by ϑ_j Assume that ϑ_j is uniformly distributed over [0,1]. The transaction frequency of the merchant market is captured by n. The merchant expects a consumer demand D_u.

For each transaction, a merchant gets a benefit b^M. As in the case of the consumer this benefit is a per-period benefit. This benefit mainly comes from the savings generated by reducing the needs to handle cash. For the passenger transportation sector in Hong Kong, processing of coin payments could cost up to 4% of the fares collected (Poon and Chau, 2001). The performance of the underlying technology will be a major factor on how much savings can be generated.

To adopt and use the micro-payment system, merchants need to install readers and terminals. Their staffs need to be trained on processing payments using the new system. All these one-time costs are represented by ϕ^M and are per-period costs. These costs are partly determined by the nature of the technology, which defines equipment requirements and user interface effectiveness.

Hence the net surplus an adopted merchant can get is:

$$V_j = D_u \vartheta_j nb^M - P_m - \phi^M$$

If a merchant does not adopt the electronic payment system, she will get the reservation utility. Without loss of generality, we assume a zero reservation utility. Here we also denote the overall system benefits as $B^M = nb^M$. Therefore, a merchant will adopt the electronic payment system if and only if

$$V_j = D_u \vartheta_j B^M - P_m - \phi^M \geq 0$$

The table below summarized the merchant parameters in the model, and its linkage to real world examples:

Table 2: Merchant side model parameters

Parameters	Description	Examples from the Octopus case
n	The transaction frequency of the merchant side of a particular market	When Octopus was launched into the public transportation market, the daily transactions valued over 2.5 million dollars, which was shared by 5 major transportation operators. Considering the average value of one dollar for a public transport transaction, each operator transacted very frequently with the public.
b^M	Merchant benefits per transaction	One major source of this saving is cost reduction. In the Octopus case, processing of coin payments could cost up to 4% of the fares collected (Poon and Chau, 2001).
ϕ^M	The cost of installing readers and terminals, training the staff, etc.	It was partly determined by the nature of the Octopus technology, which determined equipment requirements and user interface effectiveness

4. Consumer and Merchant Interaction

Suppose the marginal consumer and merchant (i.e. the consumer and merchant who is indifferent between adopting the system and staying without it) are of type $\hat{\theta}$ and $\hat{\vartheta}$. By uniform distributions of θ_i and ϑ_j:

$$D_u = 1 - \hat{\theta} \text{ and } D_m = 1 - \hat{\vartheta}$$

By simple computation, zero utilities for the marginal consumer and the marginal merchant will imply:

$$\begin{cases} D_u = 1 - \dfrac{P_u + \phi^U}{B^U D_m} \\ D_m = 1 - \dfrac{P_m + \phi^M}{B^M D_u} \end{cases}$$

The two equations are the best response functions of the two markets (consumers and merchants). Denote the consumer-side normalized cost of adoption $\dfrac{P_u + \phi^U}{B^U}$ by r_u, and the merchant-side normalized cost of adoption $\dfrac{P_m + \phi^M}{B^M}$ by r_m. Solving the above two equation gives:

$$D_m = \frac{(1 - r_m + r_u) \pm \sqrt{(1 - r_m + r_u)^2 - 4r_u}}{2}$$

$$D_u = \frac{(1 - r_u + r_m) \pm \sqrt{(1 - r_u + r_m)^2 - 4r_m}}{2}$$

In general, there are two solutions for D_u and D_m. Graphically, the two best response functions will intersect as below:

From Figure 2 we can see that there is a minimum level of consumer (merchant) acceptance required to have any positive merchant (consumer) acceptance. We call these levels the survival masses. It is interesting to see that the merchant-side (consumer-side) survival masses (i.e. the minimum required level of consumer (merchant) acceptance) is determined by the normalized cost of adoption of the merchant (consumer) market while the equilibrium merchant and consumer demands are determined by the normalized costs of adoption of both sides of the market.

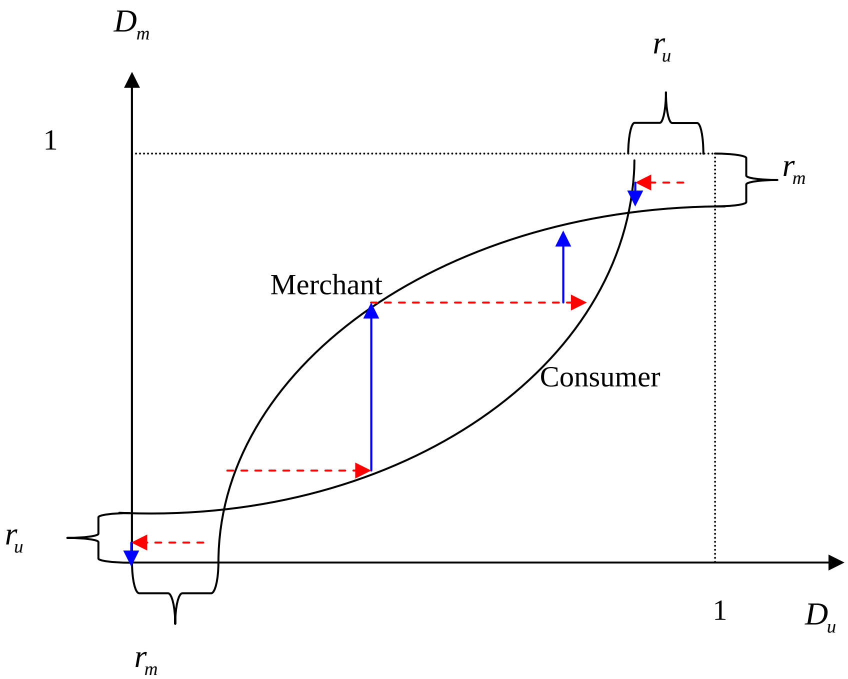

Figure 2: Interaction of Consumers and Merchants

The survival masses are the minimum consumer and merchant demands needed for a market of the micro-payment to exist. Figure 3 summarizes this phenomenon.

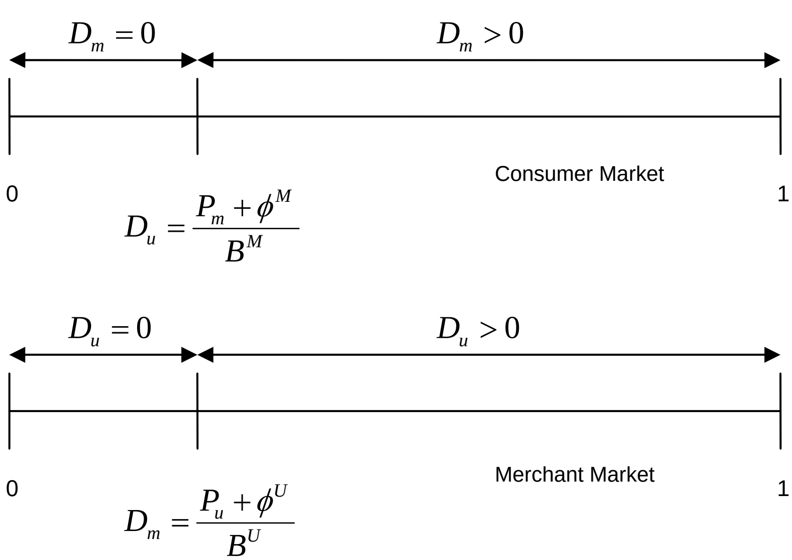

Figure 3: Survival Mass Behaviour of the Consumer and Merchant Markets

However, simply attaining the survival masses cannot guarantee a stable market. There are two intersection points of the two best response functions. Both are possible equilibrium points corresponding to low and high acceptance levels.

Checking for the stability of the two equilibria, we see that only the high acceptance level point is stable. The market will automatically go back to the high equilibrium point given any small deviations due to some shocks.

The low equilibrium point may be interpreted as the point of critical mass. A small deviation below this point will cause the acceptance levels to go to zero. If the deviation is above, acceptance levels automatically go to the high equilibrium point due to market forces. Any network size below the critical mass will have negative expectations dominant and the acceptance level tends to go to zero, while any network size above will ignite positive expectations and the acceptance level goes to a very high level – this is consistent with the process described by Shapiro and Varian (1999).

The value of having a "survival mass" as well as a "critical mass" can be best appreciated using Figure 4 below:

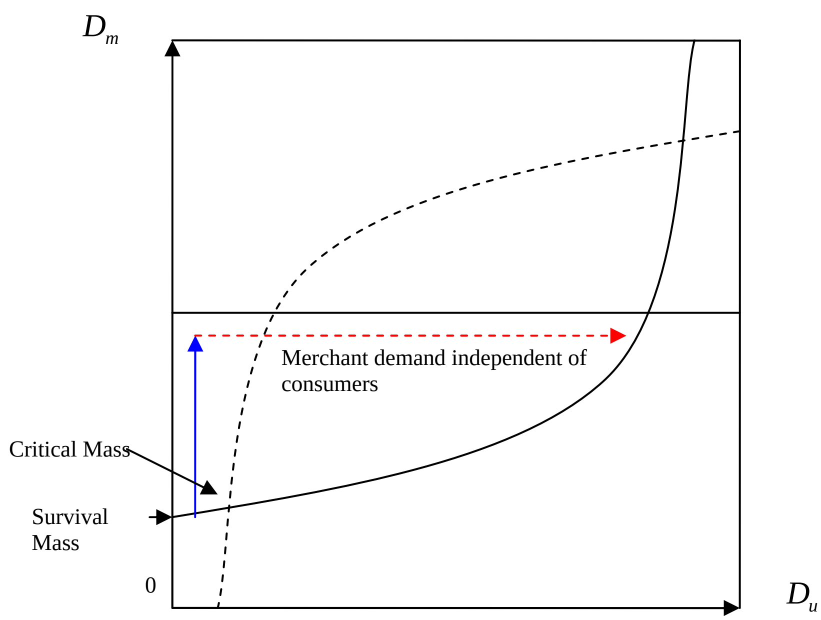

Figure 4: Critical Mass and Survival Mass

When there is only one-sided network effect, say the merchant demand is independent of the consumer demand, the merchant best response function will then be a horizontal line, as shown in Figure 4. Hence, the point of survival mass behaves as the point of critical mass which had been found by Riggins, Kriebel and Mukhopadhyay (1994).

When two-sided network effects are introduced into the market, the merchant best response function becomes the dotted curve. The point of critical mass, being the lower intersection point of the two curves, is now *a separate point* and *higher than* the point of survival mass. The intuition behind is as follows. When network effects exist only on one side of the market, the point of critical mass coincides with the point of survival mass, which is completely determined by the normalized cost of adoption. Two-sided network effects introduce extra uncertainty described by the chicken-and-egg dilemma. The market requires a certain premium to take off. Hence, the point of critical mass is separated from and is above the point of survival mass. The difference between the critical mass and the survival mass is the premium due to additional uncertainty introduced by two-sided network effects.

To summarize, a level of survival mass is required for a market to exist while a level of critical mass has to be exceeded to have a stable market. The survival mass is completely determined by the normalized cost of adoption. The critical mass is higher than the survival mass due to two-sided network effects.

The lower bound of equilibrium acceptance levels can also be established.

Proposition 1: The optimal consumer and merchant acceptance levels are both above 50%:

$$D_u^* > \frac{1}{2}, D_m^* > \frac{1}{2}$$

Proof: Please refer to appendix for proof details.

For a micro-payment system to exist (in other words, at any Nash equilibrium), the optimal acceptance levels for both sides of the market (consumer and merchant) has to be greater than 50%. This is the critical acceptance level that has to be exceeded for the market to be stable.

5. Pricing Decision in a Monopoly market

In stage one, the system provider offers prices to consumers and merchants. The system provider selects prices to maximize profit. Assume that there are only fixed costs[2] and we omit the fixed cost notation. Hence the profit function is:

$$\pi = P_u D_u + P_m D_m$$

It is widely observed that merchants have a greater incentive to adopt the micro-payment system, since the cost savings are great:

Assumption 1: Merchants, if adopting the micro-payment system, have more benefits than consumers. That is, $B^M > B^U$.

In addition, it usually costs much more for merchants to adopt the system, since it involves installing the necessary equipment and training staff to use it. The consumers on the other hand only need to get the card and learn how to use it:

Assumption 2: It costs more for merchants than consumers to adopt the micro-payment system. That is, $\phi^M > \phi^U$.

The two assumptions represent real market situations and thus have face validity.

Proposition 2: Given the conditions below:

i. $B^U > \dfrac{B^M}{2}$;

ii. $\dfrac{B^M}{\phi^M} > \dfrac{1}{2}\dfrac{B^U}{\phi^U}$;

iii. $B^U > 4\phi^M$;

there exists a unique sub-game perfect Nash equilibrium. The optimal price and acceptance levels for the consumer and merchant side of the market are given by:

$$D_u^* = \alpha$$

$$D_m^* = -\frac{1}{2}\left(\frac{B^U\alpha^2 - B^U\alpha - B^M\alpha + \phi^M}{B^M\alpha}\right) \qquad P_u^* = -\frac{1}{2}\left(\frac{B^U(1-\alpha)(B^U\alpha^2 - B^U\alpha - B^M\alpha + \phi^M)}{B^M\alpha}\right) - \phi^U$$

$$P_m^* = \left(1 + \frac{1}{2}\left(\frac{B^U\alpha^2 - B^U\alpha - B^M\alpha + \phi^M}{B^M\alpha}\right)\right)B^M\alpha - \phi^M$$

where α is the second largest root of the equation:

$$\begin{aligned} &f(x) \\ &= 3(B^U)^2x^4 - 4B^U(B^U + B^M)x^3 + ((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^M B^U - 4\phi^U B^M)x^2 - (\phi^M)^2 \\ &= 0 \end{aligned}$$

Proof: For proof details, please refer to appendix.

The interpretation of conditions i-iii is as follows. There is a market for the micro-payment system if the consumer-side system benefit is large enough relative to the merchant-side (condition (i)), the merchant side market has a high enough benefit-to-cost ratio relative to the consumer side (condition (ii)), and the consumer-side benefit, relative to the merchant-side adoption cost, is high enough (condition (iii)).

Achieving the three conditions by adjusting the model parameters is not straightforward. For instance, lowering the consumer-side adoption cost, instead of decreasing the lower bound condition requirement, will increase the lower bound requirement of condition (ii). Thus contrary to expectations, lowering the consumer-side adoption cost alone will not facilitate the establishment of the micro-payment system. Table 3 details the various effects of market parameters on equilibrium conditions.

Table 3: Impact of Market Parameters on the Conditions for Equilibrium

Conditions for Equilibrium	(i)	(ii)	(iii)
Consumer-side System Benefit (B^U)	+	-	+
Merchant-side System Benefit (B^M)	-	+	0
Consumer-side Adoption Cost (ϕ^U)	0	-	0
Merchant-side Adoption Cost (ϕ^M)	0	+	+

Proposition 3: To achieve full acceptance level for either side of the market, subsidization is required.
Proof: Please refer to appendix for proof details.

To achieve full acceptance level for either side of the market, the system provider needs to charge a negative price, i.e. subsidization is required. Hence, it is not feasible for the system provider to have full acceptance levels for both sides of the market without subsidization.

Using the Envelope Theorem, impacts of market conditions on optimal acceptance levels, prices, and profits are derived. The derivation details are in appendix and Table 4 summarizes the comparative statics.

Table 4: Comparative Statics Summary

Equilibrium Levels	D_m^*	D_u^*	P_m^*	P_u^*	π^*
Consumer-side System Benefit (B^U)	+	-	-	+	+
Merchant-side System Benefit (B^M)	-	+	+	-	+
Consumer-side Adoption Cost (ϕ^U)	0	-	-	-	-
Merchant-side Adoption Cost (ϕ^M)	-	0	-	-	-

From Table 4, Proposition 4 is observed.

Proposition 4:
1. Increasing consumer-side (merchant-side) system benefit will decrease consumer (merchant) acceptance level and merchant (consumer) price.
2. Decreasing consumer-side (merchant-side) adoption cost will have no effect on merchant (consumer) acceptance level.

Proposition 4 can be better understood if we look at the inverse demand functions:

Consumer-side: $$P_u = (1 - D_u) B^U D_m - \phi^U \quad (1)$$

Merchant-side: $$P_m = (1 - D_m) B^M D_u - \phi^M \quad (2)$$

The intuition behind is as follows: By two-sided network effects, the consumer and merchant markets are linked through corresponding acceptance levels. With a higher consumer-side system benefit, the consumer demand given by (1) is shifted upwards and becomes less elastic. Hence, a higher optimal consumer price and lower optimal consumer acceptance result. Given a lower consumer acceptance, the merchant demand given by (2) becomes more elastic. Consequently, the optimal merchant price is lower and the optimal merchant acceptance is higher. Then the higher merchant acceptance feeds back to the consumer side equation (1) and the whole process reinforce itself as illustrated in Figure 5.

Consumer-side adoption cost determines the intercept of (1). A lower consumer-side adoption cost is equivalent to an upward consumer demand shift and moves up both consumer price and consumer acceptance. Again due to two-sided network effects, the higher consumer acceptance makes possible a higher merchant price and a lower merchant acceptance through (2). This lower merchant acceptance level is fed back to the consumer side and the process illustrated by Figure 5 begins. The process stops when the decrease in merchant acceptance caused by the change in consumer-side adoption cost is erased out completely. At that point, the consumer price and consumer acceptance should be still higher than original as the effect of the consumer side demand shock will not vanish. The impact of having a higher consumer acceptance is completely captured by a higher merchant price and the final merchant acceptance level is the same as the beginning one.

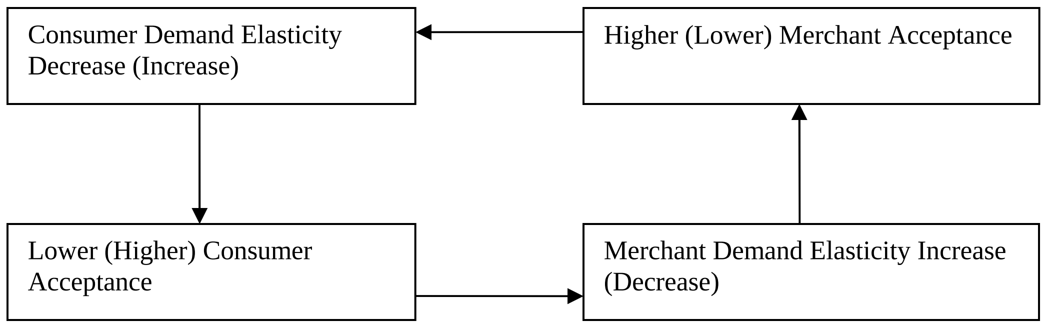

Figure 5: Interaction effects of Two-sided Market Structure

Proposition 5:
(i) The optimal consumer acceptance level is higher than the optimal merchant acceptance level.
(ii) The optimal merchant price is greater than the optimal consumer price if and only if the difference of system benefits between the two sides of the market is large enough when compared with the corresponding difference in adoption costs.
Proof: For proof details, please refer to appendix.

The asymmetric market nature (Assumption 1 and 2), together with the two-sided network effects, infers Proposition 5. With this proposition, we are able to set the priority of model parameters in terms of their impacts on profits.

Proposition 6:
1. ϕ^U has the greatest impact on the profit of the system provider;
2. ϕ^M has a higher impact on profit than B^M;
3. B^U has the least impact on the profit of the system provider.
Proof: Please refer to appendix for proof details.

Parts 1 and 3 of Proposition 6 are a direct result of Proposition 5. The relative magnitude of the impact of improving different adoption costs on profits depends on the relative network size of the consumer and merchant markets. Further, using Proposition 1 establishes Part 2 of Proposition 6. Figure 6 illustrates the relative order of impacts on profits by different market parameters.

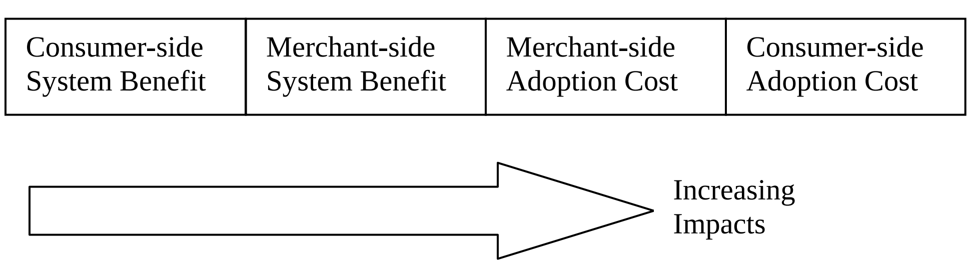

Figure 6: Order of Profit Impacts by Market Parameters

6. Discussion, Conclusion and Implication

Using a parsimonious model of two-sided network effects, we analyse the interaction of consumers and merchants and the strategy of a system provider in the context of micro-payment. Our results have important managerial insights for micro-payment system markets.

A "survival mass" of merchants and consumers is required for the market to exist and a "critical mass" for the acceptance levels to take off and remain stable. The survival mass is completely determined by the normalized cost of adoption. The critical mass is higher than the survival mass due to two-sided network effects. Managers should thus add a premium above the survival mass when they aim at getting a critical mass for the micro-payment system. Price based on cost recovery is not appropriate and may fall short in generating the critical mass for the micro-payment to exist.

There is also a lower bound, which equals to 50%, for the consumer and merchant demands. This is the critical acceptance level that has to be achieved. The Octopus card system attained acceptance levels of more than 50% on both sides of the market from its inception, while Mondex and Visa Cash fell short in achieving the 50% level before they failed and exited the market. The initial participation of the five major public transportation operators guaranteed the necessary merchant and consumer demands. The participation of the major transportation operators in the Octopus system ensured that the demands were above critical mass as a result of which Octopus turned out to be a success.

As shown in Proposition 2 and summarized in Table 3, the relationship between model parameters and the existence of a market for the micro-payment system is complicated. Improving system benefits of either consumers or merchants has a mixed effect in achieving the conditions for existence of equilibrium. Under certain conditions, improving consumer side adoption costs may even prohibit a market to exist. Independently considering improving system benefits and/or adoption costs of either market side does not help. It is the complex relationship between model parameters that matters.

Managers should be aware of this complex relationship. It is possible that when looking at one side (consumers or merchants) of the market, the micro-payment system seems to be advanced enough (having high system benefit or low adoption cost). However, when taking both sides of the market together, the conditions for equilibrium are not met. The conditions for equilibrium require the right mix of incentives for both sides of the market.

Moreover, when the conditions for equilibrium are not met, a micro-payment market does not exist at any price. That is, subsidization alone cannot create the market. This sheds lights on why micro-payment projects, like Visa Cash and Mondex that were supported by large firms did not take off. Thinking of market parameters independently and trying to create a market through subsidization is the problem. For instance, in a large scale pilot project for Visa Cash and Mondex (Van Hove, 2000), the system provider tried to get the conditions for equilibrium by subsidizing both consumers and merchants, and failed.

The comparative statics results are complex and non-intuitive. Increasing consumer-side system benefit will decrease consumer acceptance levels and merchant price while decreasing consumer-side adoption cost will have no effect on merchant acceptance levels. Similarly, increasing merchant-side system benefit will decrease merchant acceptance levels and consumer price while decreasing merchant-side adoption cost will have no effect on consumer acceptance levels. The reason behind this is the non-trivial interaction of consumers and merchants under two-sided market structure, as illustrated in Figure 5.

An increase in demand elasticity on one side of the market will lower the demand elasticity of the other side. The only way to shift optimal prices of both sides and acceptance level of one side (e.g. the consumer side) in the same direction while keeping the acceptance level of the other side (e.g. the merchant side) unchanged is to create a demand shock on the other side by adjusting the corresponding adoption cost.

There is a priority in choosing which market parameters to adjust when considering enhancing profits, as illustrated in Figure 6. In general, improving the fixed costs (the adoption costs) faced by consumers and merchants produce better profits than improving the variable benefit components (the system benefits), which partly determine the intensity of two-sided network effects. Due to the asymmetric nature of the micro-payment based market, when choosing which adoption costs to lower, the consumer side is preferred. For system benefits, it is better to increase the merchant side.

There are two main insights for subsidization and acceptance levels. The optimal merchant price is higher than the optimal consumer price under normal situations where the difference in system benefits between the two sides, as compared with that in adoption costs, is high enough. When subsidization is needed, the consumer side will be subsidized. The merchant side will receive subsidization only under very rare conditions (either when the difference in system benefits is too small, or when the difference in adoption costs is too large, or both). The Octopus system achieved its success without subsidizing either the consumer or the merchants.

To achieve full acceptance level for either side of the market, subsidization is required. Hence, it is not feasible for the system provider to have full acceptance levels for both sides of the market. In fact, no such situation is observed from real practice.

Future Direction

Our model does not consider the duopoly case. Future research may develop duopoly models to investigate competitive decisions of system providers offering different technologies. Researchers may also consider whether it is possible to have different system providers serving different market segments and if a first mover has any advantages over new entries. It will also be interesting to see if the divide-and-conquer strategy is a good one in different market structures.

Finally, researchers may further investigate the institutional question. What institutional settings will be optimal for micro-payment systems? In the credit card market, and also Mondex and Visa Cash, the payment system is owned by an independent firm. For the Octopus card, it is owned by the public transportation operators who are merchants themselves. This difference in institutional settings could be an explanation for the success of the system. In addition, the Octopus card – the most successful micro-payment system so far – operates locally in Hong Kong. The possibility of global adoption will be an interesting question. The authors expect that globalisation may require different institutional settings than local operations.

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[1] Micro-payment systems are very consumer friendly. Most of them use contact-less cards while a few use magnetic cards. These cards are very easy to learn and use for consumers. Thus there is no noticeable individual difference in efforts spent to adopt them.

[2] High fixed costs and negligible variable costs is a unique feature of information technology markets. For details, please see Shapiro and Varian (1999).

APPENDIX

Proof of Proposition 1:

The proof involves solving the response functions of the consumer and the merchant and showing that a solution exist only for the case where the equilibrium demands are greater than 0.5.

Consider the following graph:

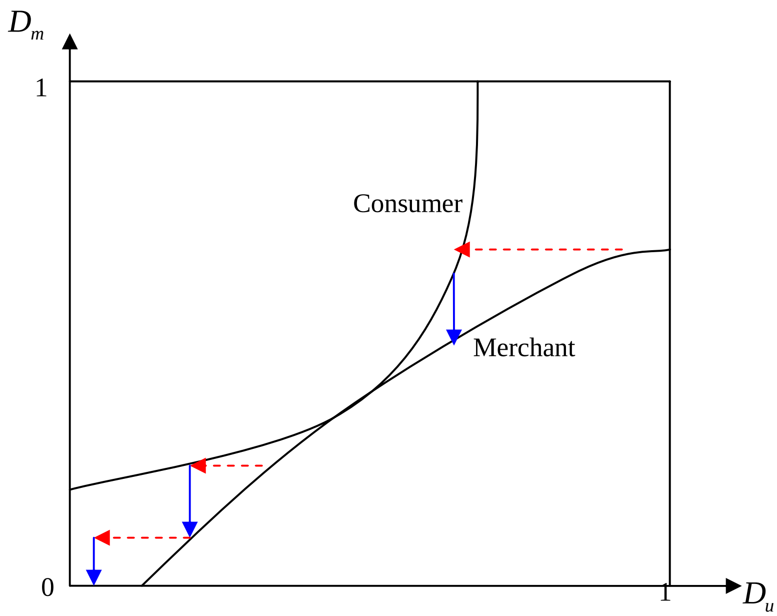

Figure 7: The Case of One Unstable Equilibrium

The above figure shows the case when the consumer and the merchant response curves touch at only one point. This equilibrium is not stable. A small shock below the equilibrium will move both the consumer and merchant acceptances to zero. Therefore, no stable equilibrium can be achieved if the two best response functions intersect at only one point. The two best response functions should intersect at two separate points and the higher intersection point will be the stable equilibrium. In addition, as shown in Figure 2, the higher equilibrium point must have both consumer and merchant acceptances higher than that of the lower equilibrium point.

The equilibrium consumer and merchant acceptance levels are given by solving the two best response functions. Consequently, optimal consumer acceptance level D_u^* is given by the larger root of the following equation:

$$f(u) = 0$$

$$= u^2 - (1 - \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M})u + \frac{P_m + \phi^M}{B^M}$$

and optimal merchant acceptance level D_m^* is given by the larger root of the following equation:

$$g(v) = 0$$

$$= v^2 - (1 + \frac{P_u + \phi^U}{B^U} - \frac{P_m + \phi^M}{B^M})v + \frac{P_u + \phi^U}{B^U}$$

To find the intersection point of $g(v)$ and $f(u)$, we set $u = v$ and get:

$$\begin{cases} u^2 - (1 - \frac{P_u + \phi^U}{B^U} + \frac{P_m + \phi^M}{B^M})u + \frac{P_m + \phi^M}{B^M} = 0 \text{..........(1)} \\ u^2 - (1 + \frac{P_u + \phi^U}{B^U} - \frac{P_m + \phi^M}{B^M})u + \frac{P_u + \phi^U}{B^U} = 0 \text{............(2)} \end{cases}$$

Subtracting (2) from (1) and simplifying the equation gives:

$$2\left(\frac{P_u+\phi^U}{B^U}-\frac{P_m+\phi^M}{B^M}\right)u=\frac{P_u+\phi^U}{B^U}-\frac{P_m+\phi^M}{B^M}$$

When $\frac{P_u+\phi^U}{B^U}\neq\frac{P_m+\phi^M}{B^M}$, it can be easily shown that $g(v)$ and $f(u)$ will intersect only at $u=v=1/2$. As both $g(v)$ and $f(u)$ are convex, both must have two roots to get the larger roots as the stable equilibrium and the two larger roots must be larger than the two smaller roots as shown by the graphical analysis, both larger roots of $g(v)$ and $f(u)$ must lie above the intersection point. That is, $D_u^{\,*}>\frac{1}{2}, D_m^{\,*}>\frac{1}{2}$.

When $\frac{P_u+\phi^U}{B^U}=\frac{P_m+\phi^M}{B^M}$, $g(v)$ and $f(u)$ become the same. Solving the equation for the larger root will give $D_u^{\,*}=D_m^{\,*}=\frac{1}{2}+\frac{1}{2}\sqrt{1-4\frac{P_u+\phi^U}{B^U}}>\frac{1}{2}$, as long as the root is real (i.e. $\frac{P_u+\phi^U}{B^U}<\frac{1}{4}$), since there must be two roots for the equilibrium to be stable as shown earlier. [QED]

Proof of Proposition 2:

We prove the proposition in 3 steps:

Step 1: Restate the profit maximization problem using the inverse demand function instead of best response functions;

Step 2: Solve the problem using first order conditions and get the 4th order polynomial $f(x)$. Two of the three second order conditions (the second partial derivatives with respect to acceptance levels) are clearly satisfied. In verifying the third second order condition, we first prove that it has positive values by showing that it is a concave function in (D_u, D_m) and its boundary points on the (D_u, D_m) plane all give positive functional values.

Step 3: Prove that only the second largest root of $f(x)$ is the equilibrium solution. Here we use Rolle's Theorem, which states that stationary points of a polynomial define the upper and/or lower bound of the value of the corresponding roots. We show that except the second largest root, all others are out of the range between 50% and 100% acceptances.

Here are the details:

Step 1: To solve for the optimal price and acceptance levels of the game, we can substitute the best response functions of the consumer side and merchant side markets into the profit function of the monopoly, and then solve for the optimal solutions. Observe that the functional forms of the best response functions are rather complicated, involving square roots of quite a number of parameters and variables. It can be expected that this will complicate the process of finding the optimal solution.

On the other hands, we see that the indirect demand functions of the two sides of the market are far simpler as shown below:

$$P_u=(1-D_u)B^U D_m-\phi^U$$

$$P_m=(1-D_m)B^M D_u-\phi^M$$

Since the setting is a monopoly one, it is mathematically equivalent to use the best response functions or the inverse demand functions in finding the optimal solution. By substituting the inverse demand functions into the profit function of the monopoly, the optimization becomes:

$$\max_{D_u,D_m}\left(\pi=((1-D_u)B^U D_m-\phi^U)D_u+((1-D_m)B^M D_u-\phi^M)D_m\right)$$

Step 2: By using the first order conditions, the following solution is got:

$$D_u^{\,*}=\alpha$$

$$D_m{}^* = -\frac{1}{2}\frac{B^U\alpha^2 - (B^U + B^M)\alpha + \phi^M}{B^M\alpha}$$

where α is the root of the equation:

$$\begin{aligned}&f(x)\\ &= 3(B^U)^2x^4 - 4B^U(B^U + B^M)x^3 + ((B^U)^2 + (B^M)^2 + 2B^MB^U + 2\phi^MB^U - 4\phi^UB^M)x^2 - (\phi^M)^2\\ &= 0\end{aligned}$$

For $D_u{}^*$ and $D_m{}^*$ to be a solution, the second order conditions are:

$$SOC1 = \frac{\partial^2\pi}{\partial D_u{}^2} = -2B^UD_m < 0$$

$$SOC2 = \frac{\partial^2\pi}{\partial D_m{}^2} = -2B^MD_u < 0$$

$$SOC3 = \frac{\partial^2\pi}{\partial D_u{}^2}\frac{\partial^2\pi}{\partial D_m{}^2} - (\frac{\partial^2\pi}{\partial D_u\partial D_m})^2 > 0$$

The first two conditions are obviously true. For the last condition, note that

$$SOC3 = 4B^UB^MD_uD_m - ((1-2D_u)B^U - (1-2D_m)B^M)^2$$

and,

$$\frac{\partial^2 SOC3}{\partial D_u{}^2} = -8(B^U)^2 < 0$$

$$\frac{\partial^2 SOC3}{\partial D_m{}^2} = -8(B^M)^2 < 0$$

$$(\frac{\partial^2 SOC3}{\partial D_u{}^2})(\frac{\partial^2 SOC3}{\partial D_m{}^2}) - (\frac{\partial^2 SOC3}{\partial D_mD_u})^2 = 48(B^UB^M)^2 > 0$$

The above concavity test infers that SOC3 is concave in (D_u, D_m). Therefore, if SOC3 is positive at the boundary values of the optimal acceptance levels, then the solution is indeed a local maximum within the boundary. Recall that the optimal levels lie between 1/2 and 1.

$$SOC3\,|_{D_u=D_m=1/2} = B^UB^M > 0$$

$$SOC3\,|_{D_u=D_m=1} = 4B^UB^M - (B^U + B^M)^2 = (B^U - B^M)^2 > 0$$

$$SOC3\,|_{D_u=1,D_m=1/2} = 2B^UB^M - B^{U^2} = (2B^M - B^U)B^U > 0 \text{ since } B^M > B^U$$

$$SOC3\,|_{D_u=1/2,D_m=1} = 2B^UB^M - B^{M^2} = (2B^U - B^M)B^M > 0 \text{ as long as } B^U > \frac{B^M}{2}$$

Hence, $D_u{}^*, D_m{}^*$ is a solution provided that $B^U > \frac{B^M}{2}$.

Step 3:Since there are four possible values for α, we proceed to prove that only one of them is the solution. Let the four solutions for $f(x) = 0$ be $\alpha1 \ge \alpha2 \ge \alpha3 \ge \alpha4$.

$$\begin{aligned}&f'(x)\\ &= 6(B^U)^2x^3 - 12B^U(B^U + B^M)x^2 + 2((B^U)^2 + (B^M)^2 + 2B^MB^U + 2\phi^MB^U - 4\phi^UB^M)x\end{aligned}$$

By solving $f'(x) = 0$, we have $x = 0$ and the following equation:

$$
\begin{aligned}
& g(x) \\
&= 6(B^U)^2 x^2 - 12B^U(B^U + B^M)x + 2((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^M B^U - 4\phi^U B^M) \\
&= 0
\end{aligned}
$$

Let $\beta 1 \geq \beta 2$ be the two roots of the above equation. Note that:

$$
\begin{aligned}
& g(0) \\
&= 2((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^M B^U - 4\phi^U B^M) \\
&> 2((B^U)^2 + (B^M)^2 + 2B^M B^U + 2\phi^U B^U - 4\phi^U B^U) \text{ as } B^M > B^U, \phi^M > \phi^U \\
&= 2((B^U)^2 + (B^M)^2 + 2(B^M - \phi^U)B^U) > 0 \text{ as } B^M > B^U > \phi^U
\end{aligned}
$$

and,

$$
\begin{aligned}
& g(\tfrac{1}{2}) \\
&= 4[8(B^M)^2 - 10(B^U)^2 + 16\phi^M B^U - 8B^U B^M - 32\phi^U B^M] \\
&= 4[8B^U B^M(\frac{B^M}{B^U} - 1 - 10\frac{B^U}{B^M}) + 16\phi^M \phi^U(\frac{B^U}{\phi^U} - 2\frac{B^M}{\phi^M})] \\
&< 0 \text{ as long as } B^U > \frac{B^M}{2}, \frac{B^M}{\phi^M} > \frac{1}{2}\frac{B^U}{\phi^U}
\end{aligned}
$$

and $g''(x) = 12(B^U)^2 > 0$, i.e., $g(x)$ is convex.

Hence, we have $\beta 1 > \frac{1}{2} > \beta 2 > 0$. By setting $g'(x) = 12(B^U)^2 x - 12B^U(B^U + B^M) = 0$, we have $x = \dfrac{B^U + B^M}{B^U} > 1$. Consequently, $\beta 1 > 1$.

By Rolle's theorem, we have $\alpha 1 > \beta 1 > 1, \frac{1}{2} > \beta 2 > \alpha 3, 0 > \alpha 4$. As a result, only the second largest root $\alpha 2$ is a possible solution. If it lies between 1/2 and 1, then it is indeed the unique solution.

To prove that, we first establish the following inequality:

$$
\begin{aligned}
& B^M - B^U > 0 > 4(\phi^U - \phi^M) \text{ as } B^M > B^U, \phi^M > \phi^U \\
& \Rightarrow B^M - 4\phi^U > B^U - 4\phi^M \\
& \Rightarrow (B^M - 4\phi^U)^2 > (B^U - 4\phi^M)^2 \text{ as long as } B^M > 4\phi^U, B^U > 4\phi^M
\end{aligned}
$$

In fact, by $B^M > B^U$ and $\phi^M > \phi^U$, $B^U > 4\phi^M \Rightarrow B^M > 4\phi^U$. Note that:

$$
\begin{aligned}
& f(\tfrac{1}{2}) \\
&= 16(4(B^M)^2 - 16\phi^U B^M - (B^U)^2 - 16(\phi^M)^2 + 8\phi^M B^U) \\
&= 16[4B^M(B^M - 4\phi^U) - (B^U - 4\phi^M)^2] \\
&> 16[(B^M - 4\phi^U)^2 - (B^U - 4\phi^M)^2] \\
&> 0 \text{ since } (B^M - 4\phi^U)^2 > (B^U - 4\phi^M)^2 \text{ as long as } B^M > 4\phi^U, B^U > 4\phi^M
\end{aligned}
$$

and,

$$
\begin{aligned}
&f(1)\\
&= (B^M)^2 - 2B^U B^M - 4\phi^U B^M + 2\phi^M B^U - (\phi^M)^2\\
&= B^M(B^M - 2B^U) - 2\phi^M \phi^U (2\frac{B^M}{\phi^M} - \frac{B^U}{\phi^U}) - (\phi^M)^2\\
&< 0 \text{ as long as } B^U > \frac{B^M}{2} \text{ and } \frac{B^M}{\phi^M} > \frac{1}{2}\frac{B^U}{\phi^U}
\end{aligned}
$$

Together with the fact that $\beta 1 > 1 > \frac{1}{2} > \beta 2$ and $\beta 1 > \alpha 2 > \beta 2$ by Rolle's Theorem, we can conclude that $1 > \alpha 2 > \frac{1}{2}$. Thus, $\alpha 2$ is the unique sub-game perfect Nash equilibrium solution for optimal consumer acceptance level D_u^*. Then the optimal merchant acceptance level D_m^* can be calculated by the equation implied by the first order conditions. Consumer and merchant prices P_u^* and P_m^* are readily available by substituting D_u^* and D_m^* into the inverse demand functions. [QED]

Proof of Proposition 3:

If consumers are completely subsidized, i.e. they are paid whatever their adoption cost is, then all consumers will join. A similar rationale applies for the merchants. More precisely, from the inverse demand functions, $P_u^* = -\phi^U < 0 \Leftrightarrow D_u^* = 1$ and $P_m^* = -\phi^M < 0 \Leftrightarrow D_m^* = 1$. [QED]

Proof of Proposition 4:

The following proof describes the impact of the model parameters on the equilibrium prices, demands and profit. The envelope theorem is used for calculating the signs as is standard practice.

By substituting $P_u = (1 - D_u)B^U D_m - \phi^U$ and $P_m = (1 - D_m)B^M D_u - \phi^M$ into $\pi = P_u D_u + P_m D_m$, we have:

$$\pi = [(1 - D_u)B^U D_m - \phi^U]D_u + [(1 - D_m)B^M D_u - \phi^M]D_m$$

By Envelope Theorem,

$$\frac{dD_u^*}{dB^U} = \left.\frac{-\partial^2\pi/\partial D_u \partial B^U}{\partial^2\pi/\partial D_u^2}\right|_{(D_u = D_u^*, D_m = D_m^*)}, \text{ and } \frac{dD_u^*}{dB^M} = \left.\frac{-\partial^2\pi/\partial D_u \partial B^M}{\partial^2\pi/\partial D_u^2}\right|_{(D_u = D_u^*, D_m = D_m^*)}$$

$$\frac{dD_u^*}{d\phi^U} = \left.\frac{-\partial^2\pi/\partial D_u \partial \phi^U}{\partial^2\pi/\partial D_u^2}\right|_{(D_u = D_u^*, D_m = D_m^*)}, \text{ and } \frac{dD_u^*}{d\phi^M} = \left.\frac{-\partial^2\pi/\partial D_u \partial \phi^M}{\partial^2\pi/\partial D_u^2}\right|_{(D_u = D_u^*, D_m = D_m^*)}$$

Hence we have the following:

$$\frac{dD_u^*}{dB^U} = \frac{1 - 2D_u^*}{2B^U} < 0 \text{ as } D_u^* > \frac{1}{2}, \text{ and } \frac{dD_u^*}{dB^M} = \frac{1 - D_m^*}{2B^U} > 0 \text{ as } D_m^* < 1$$

$$\frac{dD_u^*}{d\phi^U} = \frac{-1}{2B^U D_m^*} < 0, \text{ and } \frac{dD_u^*}{d\phi^M} = 0$$

Again by using Envelope Theorem, the followings are got:

$$\frac{dD_m^*}{dB^U} = \frac{1 - D_u^*}{2B^M} > 0 \text{ as } D_u^* < 1,\ \frac{dD_m^*}{dB^M} = \frac{1 - 2D_m^*}{2B^M} < 0 \text{ as } D_m^* > \frac{1}{2}$$

$$\frac{dD_m^*}{d\phi^U} = 0,\ \frac{dD_m^*}{d\phi^M} = \frac{-1}{2B^M D_u^*} < 0$$

By $P_u^* = (1-D_u^*)B^U D_m^* - \phi^U$ and $P_m^* = (1-D_m^*)B^M D_u^* - \phi^M$,

$$\frac{dP_u^*}{dB^U} = \frac{1}{2}(D_m^* + \frac{B^U}{B^M}(D_u^* - 1)^2) > 0 \qquad \frac{dP_u^*}{dB^M} = -\frac{1}{2}(1-D_m^*)D_m^* + \frac{1}{2}\frac{B^U}{B^M}(1-D_u^*)(1-2D_m^*) < 0$$

$$\frac{dP_u^*}{d\phi^U} = -\frac{1}{2} < 0$$

$$\frac{dP_u^*}{d\phi^M} = -\frac{1}{2}(\frac{1-D_u^*}{D_u^*})(\frac{B^U}{B^M}) < 0$$

$$\frac{dP_m^*}{dB^U} = -\frac{1}{2}(1-D_u^*)D_u^* + \frac{1}{2}\frac{B^M}{B^U}(1-D_m^*)(1-2D_u^*) < 0 \qquad \frac{dP_m^*}{dB^M} = \frac{1}{2}(D_u^* + \frac{B^M}{B^U}(D_m^* - 1)^2) > 0$$

$$\frac{dP_m^*}{d\phi^U} = -\frac{1}{2}(\frac{1-D_m^*}{D_m^*})(\frac{B^M}{B^U}) < 0$$

$$\frac{dP_m^*}{d\phi^M} = -\frac{1}{2} < 0$$

By Envelope Theorem,

$$\frac{d\pi}{dB^U} = \left.\frac{\partial \pi}{\partial B^U}\right|_{(D_u = D_u^*, D_m = D_m^*)} = (1-D_u^*)D_m^* D_u^* > 0$$

$$\frac{d\pi}{dB^M} = \left.\frac{\partial \pi}{\partial B^M}\right|_{(D_u = D_u^*, D_m = D_m^*)} = (1-D_m^*)D_u^* D_m^* > 0$$

$$\frac{d\pi}{d\phi^U} = \left.\frac{\partial \pi}{\partial \phi^U}\right|_{(D_u = D_u^*, D_m = D_m^*)} = -D_u^* < 0$$

$$\frac{d\pi}{d\phi^M} = \left.\frac{\partial \pi}{\partial \phi^M}\right|_{(D_u = D_u^*, D_m = D_m^*)} = -D_m^* < 0$$

Proof of Proposition 5:

In order to compare the equilibrium demands and prices for the consumers and the merchants we first consider a situation where the equilibrium demands and prices for the consumers and merchants are the same. Then we look at how the equilibrium demands will change with the model parameters so that we can compare the equilibrium prices and demands. Suppose that we are given an arbitrary situation where equilibrium exists. The consumer and merchant side system benefits, and consumer and merchant side fixed adoption costs are B^U, B^M, ϕ^U, and ϕ^M respectively. According to the model setup, $B^M > B^U$ and $\phi^M > \phi^U$.

Imagine that we are initially in a situation with some different merchant side system benefit and fixed adoption cost where $B^{M'} = B^U$ and $\phi^{M'} = \phi^U$. This situation satisfies all conditions required for the existence and uniqueness of equilibrium. Hence, we can reach the arbitrary situation given by adjusting the merchant side benefit level and adoption cost towards the given levels B^M and ϕ^M, and see how the optimal acceptance levels D_m^* and D_u^*, and price levels P_m^* and P_u^* changes accordingly by the comparative static results.

By symmetry between the two sides when system benefits and adoption costs are equal, we know that the optimal acceptance levels and price levels for the two sides will be equal. Now when the merchant side benefit is adjusted upwards from $B^{M'}$ to B^M, D_u^* will increase and D_m^* will decrease by the fact that $\frac{dD_u^*}{dB^M} > 0$ and

$\frac{dD_m^*}{dB^M} < 0$. When the merchant side adoption cost is moved from $\phi^{M'}$ to ϕ^M, there is no effect on D_u^* ($\frac{dD_u^*}{d\phi^M} = 0$)while D_m^* will be further decreased ($\frac{dD_m^*}{d\phi^M} < 0$).

Consequently, at the arbitrary given situation where $B^M > B^U$ and $\phi^M > \phi^U$, the optimal consumer acceptance level is greater than the optimal one for the merchant side, i.e. $D_u^* > D_m^*$.

Now let's see what happen to the price levels. By $\frac{dP_u^*}{dB^M} < 0$ and $\frac{dP_u^*}{d\phi^M} < 0$, the consumer price P_u^* will be lowered. To the merchant price, the direction of move is ambiguous. The increase in B^M will move P_m^* up ($\frac{dP_m^*}{dB^M} > 0$) while the upward move of ϕ^M will lower P_m^* ($\frac{dP_m^*}{d\phi^M} < 0$). The net result depends on the magnitude of the move in system benefit comparing to that of the move in adoption cost. Therefore, at an arbitrary situation where $B^M > B^U$ and $\phi^M > \phi^U$, $P_m^* > P_u^*$ if and only if $B^M > B^U$ is large enough when compared to $\phi^M > \phi^U$. [QED]

Proof of Proposition 6:

We first obtain the magnitude of the first derivate of the profit function with respect to the model parameters. Comparing these derivative and with some simple manipulations it is straightforward to obtain the results.

First, note that:

$$\left|\frac{d\pi}{d\phi^U}\right| = D_u^* > D_m^* = \left|\frac{d\pi}{d\phi^M}\right| > (1-D_u^*)D_m^* = \left|\frac{d\pi}{dB^U}\right|$$

$$\left|\frac{d\pi}{d\phi^U}\right| = D_u^* > (1-D_m^*)D_u^* = \left|\frac{d\pi}{dB^M}\right| > (1-D_u^*)D_m^* = \left|\frac{d\pi}{dB^U}\right|$$

Hence, part (i) and (iii) are proved. Comparing $\left|\frac{d\pi}{dB^M}\right|$ and $\left|\frac{d\pi}{d\phi^M}\right|$, we get the following:

$$\left|\frac{d\pi}{dB^M}\right| - \left|\frac{d\pi}{d\phi^M}\right| = (1-D_m^*)D_u^* - D_m^*$$

Observe that the above difference is increasing in D_u^* and decreasing in D_m^*. Since both acceptance levels lie between 1/2 and 1. The maximum difference is:

$$\left.\left(\left|\frac{d\pi}{dB^M}\right| - \left|\frac{d\pi}{d\phi^M}\right|\right)\right|_{(D_u^*=1, D_m^*=1/2)} = (1-1/2) - 1/2 = 0$$

$$\Rightarrow \left|\frac{d\pi}{dB^M}\right| - \left|\frac{d\pi}{d\phi^M}\right| < 0$$

Hence, part (ii) is also proved. [QED]