AN INVESTIGATION OF WILLINGNESS TO SPEND DYNAMICS IN SIMULTANEOUS ONLINE AUCTIONS

Mayukh Dass
Marketing Area
Rawls College of Business
Texas Tech University
mayukh.dass@ttu.edu

ABSTRACT

Willingness to spend (WTS), as defined by the amount a bidder is willing to spend in a particular auction event, is a crucial component for an auction's success. This paper investigates the dynamics of WTS of a simultaneous online auction of a specific genre of fine art called modern Indian art and compare it with the dynamics of cumulative Willingness to pay (WTP), using an innovative statistical method called Functional Data Analysis. Functional Data Analysis, which is fundamentally considered to recover the underlying WTS and cumulative WTP function curves of each bidder, is further used to examine the effects of current number of bids, current number of lots winning, pre-auction low estimate of the lots they are currently winning, bid time, and number of proxy bids on WTS and cumulative WTP dynamics. Results suggest that only current number of bids and bid time have significant positive effect on the bidder WTS, whereas only current number of bids have influence on cumulative WTP. Implications for auction house managers are further discussed in the paper.

Keywords: willingness to spend, functional data analysis, dynamic modeling, simultaneous online auctions, online fine art auctions

1. Introduction

With the growing popularity of online auctions and easy availability of detailed bidding information, we now have a greater prospect in micro-analyzing various online auction bidding behaviors in different auction settings that were not possible earlier. To a greater extent, we have capitalized on this opportunity by addressing a wide range of online auction topics, including, for example, auction formats [Jap 2002; Jap 2003; Lucking-Reiley 1999; Spann and Tellis 2006], price dynamics and forecasting [Jank and Shmueli 2006; Reddy and Dass 2006; Dass, Jank and Shmueli 2010], bidders' willingness to pay [Park and Bradlow 2005; Chan, Kadiyali and Park 2007], reference points [Dholakia and Simonson 2005; Kamins, Drenze and Folkes 2004], buyer and seller reputation [Melnik and Alm 2002], herding behavior [Dholakia, Basuroy and Soltysinski 2002], forward-looking behavior [Zeithammer 2006], bidder experience [Borle, Boatwright and Kadane 2006], bidder heterogeneity and auction design [Bapna et al. 2004]. Although these studies have expanded our understanding of online auctions, other bidder behavioral phenomena are still not examined. One such topic is bidders' willingness to spend in a simultaneous online auction.

Most of the prior researches in auctions examining bidders' purchase decision and values have focused on their willingness to pay (WTP). These studies investigated what drives bidders' WTP [Noussair, Robin and Ruffieux 2004; Chan, Kadiyali and Park 2007] and how it is different from bidders' willingness to accept (WTA) [Shogren et al. 1994; Plott and Zeiler 2005]. Interestingly, most of these studies assume that bidders are involved in purchasing only one product at a time in the auction. This assumption is not sufficient while examining bidders' purchase decisions in simultaneous online auctions where bidders typically tend to purchase more than one item simultaneously. In such cases, *Willingness to Spend* (WTS) is the most appropriate bidder metric to consider while examining success of auction houses. From the extant literature, we find examination of WTS mostly in the context of multi-product bundles [Gaeth et al. 2004]. We adopt their definition and define WTS in the context of online auctions as the amount a bidder is willing to spend in a particular auction event. In this paper, we define WTS as the total amount a bidder is required to pay at a given time period during the auction if he/ she wins all the items at the current highest bid amount where he/ she is currently the highest bidder. Therefore, by the above definition, WTS is the superset of *Willingness to Pay (WTP)* of bidders for the items they wish to purchase, as it is the sum of the WTP of all items. If a bidder is interested on only one item, then WTP = WTS. Although a bidder's WTP for an item is not disclosed by the bidder to others, the bidder's bidding pattern can be considered as a proxy for this unobserved

factor. As the auction progresses, bidders reveal more about the amount he/ she desire to pay for a particular item. In this paper, our goal is to examine the dynamics of this gradual revealing of the private value of the bidders.

One of the factors that affect bidder behavior in multi-item auctions is budget constraints [Benoit and Krishna 2001]. Park and Bradlow [2005] and Chan, Kadiyali and Park [2007] showed that budget constraints has a significant influence on how bidders choose items they want to bid, and when they bid on them. They used data from common value auctions of personal computer to illustrate this phenomenon. Such results do not hold true in case of private value auctions [Klemperer 2004] where bidders' private information is considered affiliated information. This means that a bidder's high bid value will generate higher bids from other bidders [Milgrom and Weber 1982]. As bidders have different level of attractiveness and different WTP for the items in his/ her wish list, the items are rank ordered in the bidders' bidding strategy. During the auction process, a bidder will bid on items until his/ her preset WTP. If the value of the item gets more than the bidder's WTP, the bidder may increase his/ her WTP of the current item due to influence of other bidders [Ariely and Simonson 2001; Heyman, Orhun, and Ariely 2004], may increase the WTP of the other items in his/ her wish list, or may change their consideration set and bid on a new item which was not previously considered. Further, prior studies in private value online auctions have also shown that bidders update their private value as auction progresses [Dass, Seymour and Reddy 2010] suggesting that budget constraint is not fixed in such auctions. Based these findings to our study, we conceptualize that bidders' WTS will be less influenced by budget constraints in private value auctions.

Extant literature on WTP is focused on common value single-item auctions. They suggest that if a bidder intends to purchase multiple items, summing WTP (cumulative WTP) of the interested items will give the total WTP of the bidder. This assumption may not hold in case of private value auctions as discussed above. Therefore, from the theoretical perspective, it is not only important to examine the dynamics of WTS in private value auctions, but also compare it with cumulative WTP to investigate the difference between them. This will essentially provide insight on how the relative valuation of the items in the bidders' consideration set changes and how it is affected by the auction related factors during the auction process.

From the auction house's perspective, this information is a key driver of bidder competition, which leads to higher prices and thus, profit generation for the auction firms. Therefore, reputable brick and mortar and online auction houses selling private value items, including Christie's, Sotheby's and Artnet.com generally spend a fortune before an auction event in advertising and promoting their auction item line-up to prospective buyers[1]. Understanding the dynamics of bidders' WTS during an auction is particularly important for online auction houses that conduct simultaneous auctions such as Artnet.com, SaffronArt.com and so on. In simultaneous auctions, a large number of heterogeneous items are sold *concurrently* to the same group of bidders over an extended period of time. Compared with single-item auctions held at eBay or Ubid.com, simultaneous online auctions give rise to a unique competitive environment as bidders participating in these auctions typically bid on one or more items at the same time. As individual bidders have unique budget constrained or depth of pocket, and desire to purchase a unique set of item/s at a specific price level, the underlying distribution of WTS varies widely among them. In this paper, we investigate the dynamics of WTS of a simultaneous online auction of a specific genre of fine art called modern Indian art using an innovative statistical method called Functional Data Analysis and compare it with the dynamics of cumulative WTP of all the items the bidders are bidding.

From the modeling perspective, online auction data present challenges that make application of traditional econometric/ regression methods difficult. The data consist of records of bid sequence placed at unevenly spaced intervals, thus making traditional time series methods inappropriate for analysis. Functional Data Analysis (FDA) [Ramsay and Silverman 2005], which at its core is the analysis of curves rather than points, is well suited to analyze this type of data. FDA is an emerging statistical methodology that operates on functional observations such as the WTS curves in online auctions. While it has received a lot of enthusiasm within the statistics literature, it is only slowly entering the e-commerce and information systems literature. In online auctions, FDA has been shown to be useful as a graphic tool for advanced data visualization of electronic commerce data [Jank et al. 2008] and as a mechanism to capture price dynamics in online auctions [Bapna, Jank and Shmueli 2008; Jank and Shmueli 2006; Reddy and Dass 2006]. Using this technique, we analyze the WTS and cumulative WTP dynamics (velocity and acceleration) in online auctions after recovering the underlying WTS and cumulative WTP curves of each bidder using a non-parametric curve-fitting technique such as splines [Simonoff 1996]. Whereas the traditional regression methods are useful in modeling the final amount spent by bidders in the auctions, FDA provides the required tool to model the WTS and cumulative WTP dynamics and determine their relationship with the strategic variables during the entire auction. The underlying idea is to represent the bidders' WTS/ cumulative WTP path during an auction as a continuous curve. Then, following functional principles, we "recover" (i.e., estimate) the WTS/ cumulative WTP

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[1] Auction event managers from Artnet.com, Christie's and Sotheby's were interviewed prior to this study.

curves of individual auctions using smoothing techniques [Ramsay and Silverman 2005]. From these functional curves, we can then obtain estimates of the WTS/ cumulative WTP dynamics such as their velocity and acceleration via first and second derivatives of the WTS/ cumulative WTP curves, respectively. Finally, we model the heterogeneity of these WTS/ cumulative WTP paths using three bidder behavior characteristics including current number of bids, bid time, and number of proxy bids and two bidder preference characteristics including current number of lots[2] winning and pre-auction low estimate of the lots they are currently winning, to provide insights on their relationship with the WTS/ cumulative WTP dynamics of bidders during the auction. To this end, this paper, to best of our knowledge, is the first to investigate WTS dynamics of an online auction of any format, compare it with the dynamics of cumulative WTP, and examine how various bidder and bidding specific factors affect the WTS and cumulative WTP differently during auction.

Functional Data Analysis (FDA) is used in this study for two reasons. First, as bids arrive in auctions at irregular time, traditional time series modeling or regression will be inappropriate as different bidders have different rate of bidding as mentioned earlier. Therefore, we need some form of a modeling technique, which can overcome the irregularity issue and interpolate the missing data. This issue is resolved using FDA where a polynomial of order n is considered to perform the curve fitting. Second, we want a technique that will allow us to investigate the effects of determinants on the WTS/ cumulative WTP dynamics during the auction process. FDA technique allows computing higher order derivatives of the WTS/ cumulative WTP and provides a structure to examine determinants' effects throughout the auction process. FDA is the term given to the overall approach of analyzing data, which includes 1) a curve fitting exercise to discover the underlying function and 2) functional regression that includes running multiple regression on the discovered dynamic function for every auction period; and the results are presented in the form of illustrating estimates obtained from the regressions at each time period.

We examine bidders' WTS/ cumulative WTP in simultaneous online auction of modern Indian art collected from a prominent online auction house called SaffronArt.com. Analysis of online auctions of hedonic heterogeneous products, like fine art, also adds another level of complexity in our analysis. First, the uniqueness of the artwork and the artists provide a modeling challenge as we are dealing with high variability across the auctioned lots. Second, these art items possess a more private (hedonic) value to the bidders than a common value. Third, a small group of dedicated art collectors and investors follow online art auctions. Therefore, the bidders who participate in these auctions are similar to the ones attending live auctions organized by Christie's and Sotheby's. Although our study can easily be replicated with data from a more prominent online auction such as EBay, current data availability and data features from the site pose a problem. EBay have stopped revealing bidder identification on their sites. Therefore, even if a bidder is bidding on multiple items simultaneously, there is no way to identify their activity across items.

In sum, the intended contribution of our study is twofold. First, we examine the willingness to spend dynamics of bidders in a simultaneous online auction setting and compare it with their cumulative willingness to pay dynamics. Second, we use a very innovative modeling technique called Functional Data Analysis to examine the dynamics. Using this technique further allows us to micro analyze the effects of three bidder behavior characteristics including current number of bids, bid time, and number of proxy bids and two bidder preference characteristics including current number of lots winning and pre-auction low estimate of the lots they are currently winning, on the WTS/ cumulative WTP dynamics of bidders during the auction. The rest of the paper is presented as follows. First, we discuss willingness to spend in the context of online auctions and discuss how it is different from WTP. Second, we introduce Functional Data Analysis and illustrate how it can be used to examine WTS/ cumulative WTP dynamics. Third, we discuss the setting of the study, i.e. modern Indian art online auctions. Fourth, we present the results from our investigation of WTS dynamics and compare them with that from cumulative WTP. We conclude by discussing the managerial implications of our work and directions for future research.

2. Bidder's Willingness to Spend

We define bidders' WTS at auction time t as the sum of highest bid of all lots the bidder is currently the highest bidder at time t. Empirically, WTS of bidder i at auction time t is measured as:

$$WTS_{it} = \sum_{l=1}^{L_{it}} WB_{lt} \text{ --- (i)}$$

Where WB_{lt} is equal to the winning bid of lot l at time t and $l=1...L_{it}$ are the lots where bidder i is currently the highest bidder. To illustrate this measure, let us consider the two bid history shown in Figures 1a and 1b. They both are bid history screenshots takes from the modern Indian art online auction conducted by SaffronArt.com during

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[2] In art auctions, each art item is called a "lot." We will use the term "lot" throughout the paper.

December 5-8 2005. The first snapshot is the bid history of lot number 39, which is a painting by a popular Indian artist called "Ram Kumar." The painting is untitled, made with *Acrylic on canvas*, has 32.5 in. by 36.5 in. dimension and has an estimated price of $55,000-$65,000. The second bid history snapshot is that of lot number 42, which is painted by another popular Indian artist called "Francis Newton Souza." This painting is titled *Tortured Indian Woman*, made with *Oil on Masonite*, has 30 in. by 24 in. dimension, and has an estimated price of $80,000-$90,000. If we want to know the WTS of a bidder called *Kyozaan* who was bidding on both these items concurrently at 10:00 AM on December 6, 2005, it will be $121,500 (the winning bid from lot#42). In other words, if no one else bid on these two items after this point in time, *Kyozaan* is required to pay the WTS amount to the auction house as he/ she will win both these items. WTS links directly to the bidders' budget constraints. It not only suggest the depth of pocket of a particular bidder, but its dynamics also reveal the strategies a bidder follow to optimize the number of items they win and the amount they are willing to spend.

The following are the bids that have been placed on this Lot : COUNTDOWN : 1 days 22:29:35

❖39
Ram Kumar

Untitled

Signed and dated in English (reverse)
2004
Acrylic on canvas
32.5 x 36.5 in (82.6 x 92.7 cm)

$55,000 - 65,000
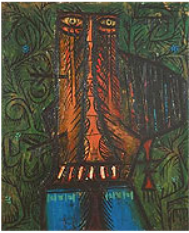Rs 2,420,000 - 2,860,000

Next Valid Bid : $ 76,500 (Rs 3,366,000)

Rank	Nick Name	Amount($)	Amount(Rs)	Bid Type	Date & Time (US EST)
Current Highest Bid 1	Poker	71,500	3,146,000		Dec 6 2005 3:03:48 AM
2	Kyozaan	66,500	2,926,000		Dec 6 2005 1:27:43 AM
3	Poker	61,500	2,706,000		Dec 6 2005 1:26:17 AM
4	Kyozaan	56,500	2,486,000		Dec 6 2005 1:26:10 AM
5	Anonymous 3	51,500	2,266,000	Proxy	Dec 6 2005 1:25:58 AM
6	Kyozaan	49,000	2,156,000		Dec 6 2005 1:25:58 AM
7	Anonymous 3	46,500	2,046,000	Proxy	Dec 5 2005 10:30:00 PM
	Start price	44,000	1,936,000		Dec 5 2005 10:30:00 PM

Figure 1a: Bid history from Simultaneous Online Modern Indian Art Auction

The following are the bids that have been placed on this Lot : COUNTDOWN : 1 days 22:30:53

❖42
Francis Newton Souza

Tortured Indian Woman

Signed and dated in English (upper right and reverse)
1957
Oil on masonite
30 x 24 in (76.2 x 61 cm)

$80,000 - 90,000
Rs 3,520,000 - 3,960,000

Next Valid Bid : $ 131,500 (Rs 5,786,000)

Rank	Nick Name	Amount($)	Amount(Rs)	Bid Type	Date & Time (US EST)
Current Highest Bid 1	Kyozaan	121,500	5,346,000		Dec 6 2005 9:53:24 AM
2	socrates	111,500	4,906,000	Proxy	Dec 6 2005 9:52:25 AM
3	Kyozaan	101,500	4,466,000		Dec 6 2005 9:52:25 AM
4	socrates	94,000	4,136,000	Proxy	Dec 6 2005 4:42:35 AM
5	Poker	86,500	3,806,000		Dec 6 2005 4:16:25 AM
6	Anonymous 38	79,000	3,476,000		Dec 6 2005 4:14:37 AM
7	Kyozaan	74,000	3,256,000		Dec 6 2005 1:16:56 AM
8	Anonymous 38	69,000	3,036,000		Dec 5 2005 10:40:07 PM
	Start price	64,000	2,816,000		Dec 5 2005 10:30:00 PM

Figure 1b: Bid history from Simultaneous Online Modern Indian Art Auction

With WTS considered mostly as a superset of WTP, most of the extant literature on auctions has investigated WTP at different auction settings. This literature body can be broadly grouped into two types of studies, with one modeling and estimating bidders' WTP and the other examining the factors effecting WTP. Examples of papers related to the first type are Park and Bradlow [2005] who modeled bidders' WTP in single item online auction of computers. Chan, Kadiyali and Park [2007] also looked at consumer willingness to pay (WTP) from an English or ascending first-price auction based on two general bidding premises: no bidder bids more than his/ her WTP, and no bidder allows a rival bidder to win at a price that he/ she is willing to beat. Recently, Goes, Karuga and Tripathi [2009] investigated how bidders update their WTP in sequential auctions and what type of information influence this process. The second type of studies have looked at the micro auction factors such as auction setting [Noussair, Robin and Ruffieux 2004; Roth and Ockenfels 2002] and seller reputation [Melnik and Alm 2002] affecting bidders' WTP. Our study complements this body of literature by investigating the changes in WTS and comparing it with the changes in cumulative WTP of a bidder at each time period during the auction. In particular, we examine a scenario where bidders are interested in purchasing more than one item simultaneously. It also contributes to the understanding of the effects of three bidder behavior characteristics including current number of bids, bid time, and number of proxy bids and two bidder preference characteristics including current number of lots winning and pre-auction low estimate of the lots they are currently winning, on WTS and cumulative WTP dynamics. We compute cumulative WTP of bidder i at auction time t as:

$$WTP_{it} = \sum_{q=1}^{Q_{it}} HB_{qt} \quad \text{--- (ii)}$$

where HB_{lt} is equal to the highest bid of lot q at time t and $q=1...Q_{it}$ are the lots where bidder i is currently bidding on or has bid earlier. To illustrate this measure with the two bid histories (Figure 1) discussed earlier, consider computing Kyozaan's cumulative WTP value at 10:00 AM on December 6, 2005. In this case, it will be $188,000 ($66,500 from lot#39 +$121,500 from lot#42).

3. Determinants of Willingness to Spend

The research framework of our study is illustrated in Figure 2 and the explanatory variables are discussed below.

3.1. Current Number of Bids

Current number of bids of a bidder indicates the level of activity of a bidder [Reddy and Dass 2006]. If a bidder has a high bidding activity in the auction, it is possible that either (1) the bidder prefers incremental bidding or (2) the bidder is updating his/ her WTP with information learned during auction [Goes, Karuga and Tripathi 2009]. In either case, the WTS of the bidder is expected to change over time with number of bids. Therefore, we expect the number of bids to have a positive effect on the WTS and cumulative WTP dynamics.
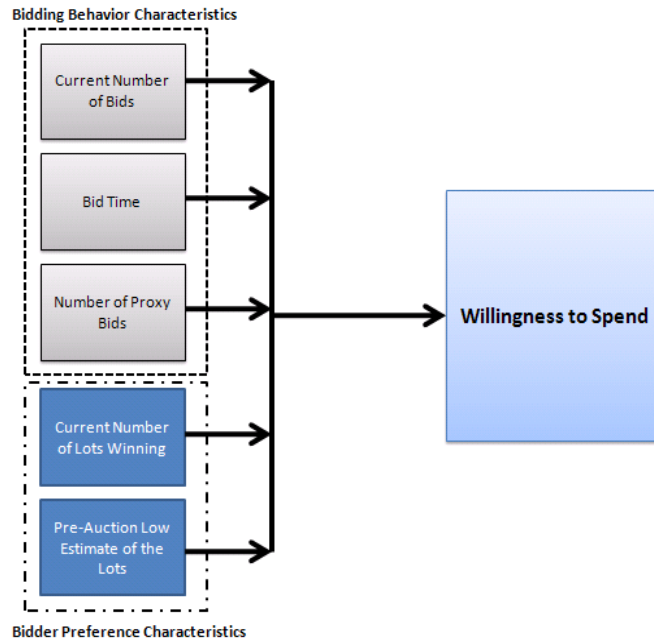

Figure 2: Research Framework for Willingness to Spend

3.2. Bid Time

Average bid time of bidders also play a significant role in the WTS/ cumulative WTP change of the bidders. If a bidder joins early in the auction, their level of WTS / cumulative, WTP will be low, but if he/ she joins late, their level of WTS/ cumulative WTP may be comparatively high. Therefore, we expect that with increase in bid time, the WTS/ cumulative WTP will also increase, thus suggesting that bid time will have a positive effect on the bidders' WTS and cumulative WTP.

Although it is appropriate to assume that bidders may maintain a fixed WTP for items during the auction, prior studies have shown that this assumption is not always true and that the auction mechanism influences the upper limit of the WTS. Ariely and Simonson [2001] was the first to explicitly identify influence of one bidder over another, Heyman, Orhun, and Ariely [2004] termed this psychological phenomenon as "auction fever" and showed that wealth set for purchasing during auction is not fixed, and Dass, Seymour and Reddy [2010] modeled this phenomenon and measured how much such changes occur. Furthermore, having a high number of bids (his/ her level of commitment to the product) does not guarantee a bidder to be the leader in the bidding process. Similarly, bid time indicates his/ her activity period (how long the bidder is involved in the auction) which also plays a significant role on how much one can participate. Therefore, both these factors may influence WTS.

3.3. Number of Proxy Bids

Like eBay auctions, simultaneous online auctions also use a "proxy-bid" system, where the bids are automatically updated on behalf of the bidders. Proxy bidding is a commonly available feature in most online auction houses where bidders set a maximum amount they are willing to pay for the auctioned item and let the auction house place proxy bids on their behalf until that price. Bidders using this facility have a pre-determined value for the item and use it to stay within that value limit [Bapna et al. 2004]. Greater the number of proxy bids, less dynamic will be the bidders' WTS and cumulative WTP. Therefore, we expect the number of proxy bids of the bidders to have a negative effect on the bidders' WTS and cumulative WTP.

3.4. Current Number of Lots Winning

Bidders bidding on a large number of lots suggest that the bidder has a deep pocket, and he/ she is capable of spending a lot of money. Effect of the number of lots bid on the WTS/ cumulative WTP dynamics will depend on the level of activity in these lots. A high level of lot specific activity, i.e. large number of bids and growing number of bidders bidding, is expected to lead to large amount of changes in WTS and cumulative WTP. Interestingly, prior literature have found insignificant effect of these two factors on the price dynamics [Reddy and Dass 2006; Dass and Reddy 2008]. Therefore, following the findings of the extant literature, we expect the effect of current number of lots bid by bidders on their WTS and cumulative WTP to be insignificant.

3.5. Pre-Auction Low Estimates

Finally, pre-auction estimates play a very significant role in the auctions of high-end items such as fine art. These values are released by the auction house and are computed by their art experts and appraisers. Prior studies Czujack and Martins [1996] found that these pre-sale estimates are a good predictor of the final price. In other words, this piece of information plays an important role in the price formation and realization of the auctioned items. Therefore, an item with higher pre-auction estimate will attract comparable bids with similar price level. As the auction house releases an upper and lower bound of the estimated value, we will only consider the lower bound[3] of the pre-auction estimate of the artwork for our analysis. Therefore, we expect the pre-auction low estimate of the items a bidder is bidding on to have a positive effect on his/ her WTS and cumulative WTP.

4. METHODOLOGY

4.1. Functional Data Analysis

In general, the underlying approach to our study is to first estimate the WTS function of each bidder for each auction time period, and then use this function as a dependent variable in a regression model (called functional regression as the dependent variable is a function) to determine how different covariates affect WTS at each time period. One of the fundamental phenomenons that are captured by WTS is the changing private values of bidders based on the value information signaled by other bidders. We can only realize this changing private value when the bidder posts a bid. Since every bidder has its own bidding behavior, bids posted are non-uniform (or irregularly spaced). For example, bidders do not bid at uniform time, and therefore bids arrive at irregular time periods. Therefore, as a first stride towards our goal, we must smooth the WTS/ cumulative WTP data of bidders shown in Figure 1. This process goes through some pre-processing steps. First, we standardize the 3-day auction time by scaling it within 0 to 1, thus $0 \leq t_{ij} \leq 1$, i=1,2 ….N, where t_{ij} represents the time when WTS/ cumulative WTP of

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[3] We reanalyzed with pre-auction high estimate and did not found any significant difference in the results. The results are available upon request.

value j is recorded for bidder i and N represents the # of bidders participating in the auction. We accommodate the irregular spacing of WTS/ cumulative WTP (due to non-uniform bid arrivals) by linearly interpolating the raw data and sampling it at a common set of time points. For each of the auction time period, we compute the corresponding log transformed WTS values of the bidder to reduce the skewness of its distribution.

After the data preprocessing, the second stage in our analysis is to use penalized smoothing splines [Ramsay and Silverman 2005; Simonoff 1996] to recover the underlying WTS/ cumulative WTP curves. These curves represent the WTS/ cumulative WTP dynamics, capture the local variation in the dataset and readily provide different derivatives of the smoothed WTS/ cumulative WTP curves. This functionality allows us to analyze higher-order functions of the bidders' WTS/ cumulative WTP, namely WTS-velocity/ cumulative WTP-velocity (1st order derivative) and WTS-acceleration/ cumulative WTP-acceleration (2nd order derivative). To recover the underlying WTS/ cumulative WTP curve, we consider a polynomial spline of degree p.

$$f(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 ... + \beta_p t^p + \sum_{l=1}^{L} \beta_{pl} \left[(t - \tau)_+ \right]^p \quad ----- \text{(iii)}$$

where $\tau_1, \tau_2 ... \tau_L$ is a set of L knots and $u_+ = u I_{[u \geq 0]}$. The choice of L and p determines the departure of the fitted function from a straight line with higher values resulting in a rougher f. This may result in a better fit but a poorer recovery of the underlying trend, as it has a tendency to overfit the given data. To avoid this issue, a roughness penalty function of the following is imposed to measure the degree of departure from the straight line

$$PEN_m = \int \left[D^m f(t) \right]^2 dt \quad ---- \text{(iv)}$$

where $D^m f$, $m = 1, 2, 3 ....$, is the mth derivative of the function f. The goal is to find a function $f^{(j)}$ (WTS/ cumulative WTP) value of j for bidder i) that minimizes the penalized residual sum of squares

$$PENSS_{\lambda,m}^{(j)} = \sum^{n} \left(y_i^{(j)} - f^{(j)}(t_i) \right)^2 + \lambda \times PEN_m^{(j)} \quad ---- \text{(v)}$$

where $y_i^{(j)}$ represents the observed WTS/ cumulative WTP of value j of bidder i and $f^{(j)}(t_i)$ represents the corresponding value obtained from the computed WTS function of bidder i, the smoothing parameter λ provides the trade-off between fit $[(y_i^{(j)} - f^{(j)}(t_i))^2]$ and variability of the function (roughness) as measured by PEN_m. We used the b-spline module developed by Ramsay [Ramsay 2003] for minimizing $PENSS_{\lambda, m}^{(j)}$. The WTS function for all the 256 bidders during the auction is shown in Figure 3. As figure 3 suggests that WTS of bidders are non-monotonic in nature, a b-spline is the best possible approach.
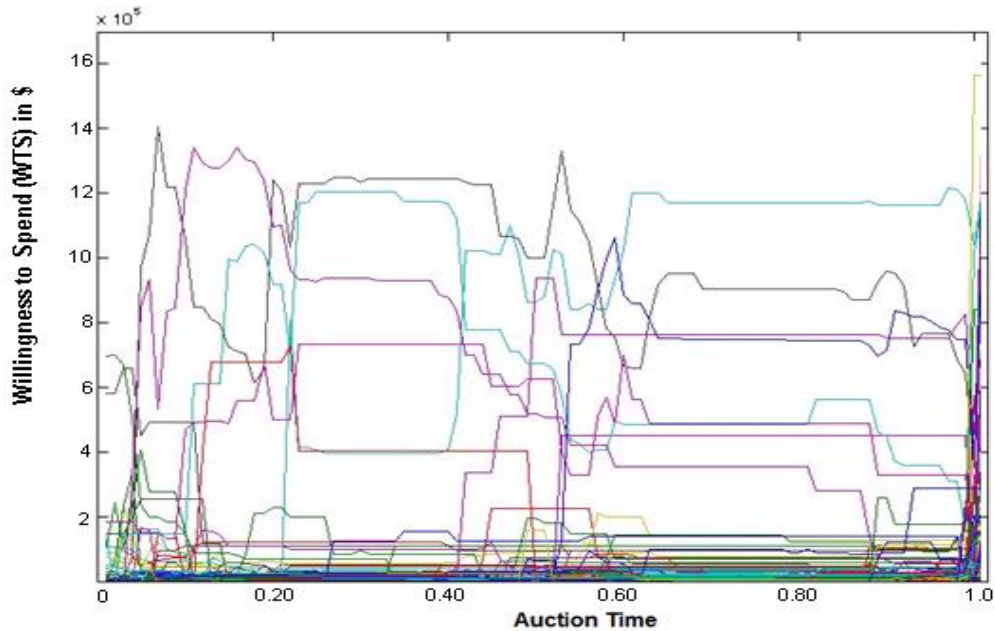

Figure 3: Willingness to Spend For Bidders in the Auction

Finally, to analyze effects of different covariates on WTS dynamics, we apply functional regression with the WTS functions as our response variable and the bidder specific determinants such as current number of bids, current number of lots bid, pre-auction low estimate of the lots they are bidding on, current average bid time, and current number of proxy bids as our explanatory variables. We now describe the formulation of a functional regression model. Let

$$Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ . \\ . \\ y_i(t) \end{bmatrix} \quad \text{--- (vi)}$$

be an N x 1 vector of functional variables where N is the number of bidders. If we want to model the current WTS, then $y_j(t) = f_j(t)$. We set $y_j(t) = f_j'(t)$ if we are modeling WTS velocity. A general linear model of the following form is considered

$$Y(t) = \beta_0 + \beta_1 \times (\text{Current \# of Bids})_t + \beta_2 \times (\text{Current \# of Lots Bid})_t + \beta_3 \times (\text{Pre-auction estimates of lots bid})_t$$
$$+ \beta_4 \times (\text{Current avg. bid time})_t + \beta_5 \times (\text{Current \# of Proxy Bids})_t + e \quad \text{---- (vii)}$$

where e is the error term of the regression model, having a normal distribution of mean 0 and variance σ^2. $\beta's$ are the parameters, which measures the influence of the covariates at every point in time. Functional regression models then allow us to understand the influence of covariates on WTS dynamics over time. As Ramsay and Silverman [2005] point out, this is achieved by estimating $\beta(t_i)$ for a finite number of points in time t (in our case t=100) and constructing a continuous parameter curve by simply interpolating between the estimated values $\hat{\beta}(t_1) \ldots \hat{\beta}(t_n)$.

To capture the effects of the explanatory variables on each of the WTS dynamic variables, we run a regression for each time period (1-100) for data from all the lots. The parameter estimates associated with each explanatory variable are plotted along with confidence bands to indicate the impact and its significance over the entire auction. A flow chart of the above method is shown in Figure 4.
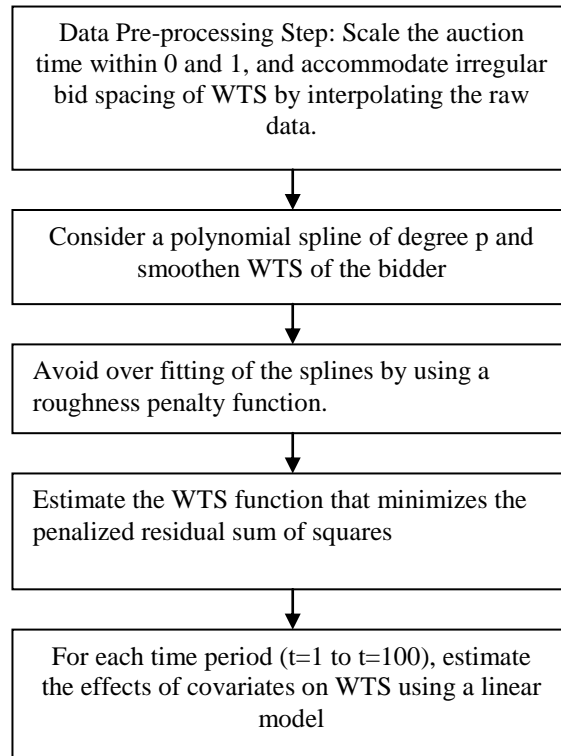

Figure 4: Flowchart of Functional Data Analysis

FDA is based on smoothing and different smoothing parameters can surely lead to different estimates. Therefore, a common approach for investigating the sensitivity of smoothing is to conduct a robustness study, that is, investigate different smoothing parameters (different knots, different penalty parameters, etc) [Ramsay and Silverman 2005; Wang, Jank, Shmueli and Smith 2008]. Sensitivity tests were also performed with different values of p (3, 4, 5, 6 were used) and λ (14 different values between 0.001 and 100 were used). We found the model fit to be insensitive to different values of p and λ. However, the RMSE for the model was the lowest with $p=4$ and $\lambda = 0.1$ for both WTS and cumulative WTP. Further, # of knots used in equation (iii) varied from bidder to bidder based on the number of instance of their WTS/ cumulative WTP change during the auction, similar to the approach used by Jank and Galit [2006].

4.2. Data

To investigate WTS/ cumulative WTP dynamics in simultaneous online auction, we collected data from an online auction house called SaffronArt.com, which sells only modern Indian art. The auctions are held over a multi-day period (typically three days). Bid histories of the auctioned items are available from the auction house's website during the auction. Figure 1 illustrates a snapshot of a bid history from the auction website. This auction uses an ascending-bid format with a soft closing time and date set by the auction house. To discourage snipping behavior of the bidders, the auction adds three more minutes to the time clock if the last bid is placed in the last three minutes of the auction[4][Roth and Ockenfels 2002]. The auction also uses a "proxy-bid" system similar to the one used by eBay, where the bids are automatically updated on behalf of the bidders. Proxy bidding is a commonly available feature in most online auction houses where bidders set a maximum amount they are willing to pay for the auctioned item and let the auction house place proxy bids on their behalf until that price. Bidders using this facility have a pre-determined value for the item and use it to stay within that value limit [Bapna et al. 2004]. For our study, we collected bidder behavior specific factors including, what items they are bidding on, when they are bidding, for how much and how many times they are bidding on.

Like other online auction houses, SaffronArt.com posts detailed bid histories of items for sale on their website during periods when auction is in progress. The bid history includes information on each submitted bid, its time, amount and the bidder's ID. Apart from the bidding activity information, each bid history also includes information about the item: name of the artist, physical characteristics of the item (size and media), pre-auction estimates, and the item's expected value based on analysis by auction house art experts, and provenance of the item. The auction house also provides information about auction results of previously sold comparable items by the same artist. Since the items are of high value, the auction house tries to provide as much information about the items as possible in order to help bidders make rational bidding decisions. Additional information about the auction format, general bidding rules, and the closing schedules is also provided by the auction house.

SaffronArt.com organizes anywhere from three to five auction events per year. In each of these events, they typically sell 100 to 200 contemporary Indian paintings and sculptures. Although the bid history is posted online *during* the auction, it is promptly removed as soon as the auction closes and there is no way of revisiting it afterwards (unlike eBay where information is stored for 15 days after an auction closes). This policy complicates the data collection task. To overcome this obstacle, we developed a java-based web agent that visits the bid history during the very last moments of the auction and that captures the entire information automatically. Since the auction is extended after a late bid, our web agent is able to capture all of the bid history without losing any information.

The particular auction we use for our investigation was held in December 5-8, 2005 where 199 lots (each lot typically is a unique piece of art – namely a painting, a drawing or a sculpture) were auctioned over a three-day period. In this particular auction, works of seventy artists were auctioned, with an average three lots per artist. The average realized price per lot was $62,065 and ranged from a low of $3,135 to a high of $1,486,100. Overall, 256 bidders participated in this online auction and placed 3080 bids. Number of bids per lot averaged 15.47 and ranged from 2 to 48. On average, six bidders participated in each lot, which ranges from 2 to 14 bidders across the auction. Mean bids per bidders were 4.93 bids, with a range of 1 to 65 bids. Some of the key descriptive information about the auction is presented in Table 1.

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[4] Roth and Ockenfels [2002] examined the difference in the last-minute bidding strategies of bidders in these types of auctions and in eBay auctions where the auction ends exactly at a particular time.

Table 1: Summary Data Description

	Mean (SD)	Median	Min.	Max.
Opening Bid in $	$19,145 ($36,830)	$6,400	$650	$300,000
Pre-Auction Low Estimates of the Lots	$23,880 (45,954)	$8,000	$795	$375,000
Pre-Auction High Estimates of the Lots	$30,816 (60,676)	$10,230	$1,025	$475,000
Realized Value of the Lots in USD($)	$61,845 (134,109)	$21,400	$3,135	$1,486,100
Realized Sq. Inch Price of the Lots in USD($)/ Sq. Inch	$109.39 (227.13)	$45.06	$1.40	$1,865.42
No. of Unique Bidders/ Lot	6.38 (2.47)	6	2	14
No. of Unique Lots Bid / Bidder	4.89 (7.76)	3	1	63
No. of Bids/lot	15.52 (7.49)	15	2	48

5. RESULTS

The WTS of bidders during the auction (Figure 3) shows a wide variation among bidders in the auction. Particularly, we see four types of WTS curves where (1) bidders demonstrate a very high WTS early in the auction, which gradually declines to a lower level as the auction progresses, (2) bidders demonstrate a uniform WTS level throughout the auction, (3) bidders show a low WTS early in the auction, which steadily increases to a higher level during the auction, and (4), a variation of type (3) with the WTS increases abruptly near the end of the auction. There three curves suggest that there are at least three prominent types of bidding strategies are being used in this auction. Bidders demonstrating the first type of curve bid heavily on multiple lots concurrently early in the auction, but as the auction progresses, other bidders top them with higher bids. The second curve suggests that these bidders are disciplined, bid on few items they are interested in and put a very high value where other bidders are unable to top it. The third curve suggests that these bidders progressively learn about their competition and the items they are willing to purchase, and gradually increase their WTS. Finally, the fourth curve indicates that there are some bidders who wait until the last few moments of the auction to place a bid. We observe very similar velocity (1st derivative) curves for bidders WTS as shown in Figure 5a. Interestingly, the WTS acceleration (2nd derivative) suggest that most significant changes in WTS occurred near the end of the auction, as shown with steep curve after 0.9 auction time (approximately 7 hours left in the auction) in Figure 6a. On examining the cumulative WTP velocity (Figure 5b), we find that there are high activity both at the beginning and at the end of the auction in comparison to WTS. This finding is further supported by the cumulative WTP acceleration (Figure 6b) plot. This difference between WTS and cumulative WTP is attributed to the way the two measures are defined. WTS only considers items where the bidder is currently winning, whereas cumulative WTP considers all items the bidder has bid. Both the velocity and acceleration of cumulative WTP indicates that there are quite a few bidders who have bid on multiple items from the beginning of the auction. As the WTS plot shows a low activity during the same period, it suggests that these bidders are not the winning bidders at that time.
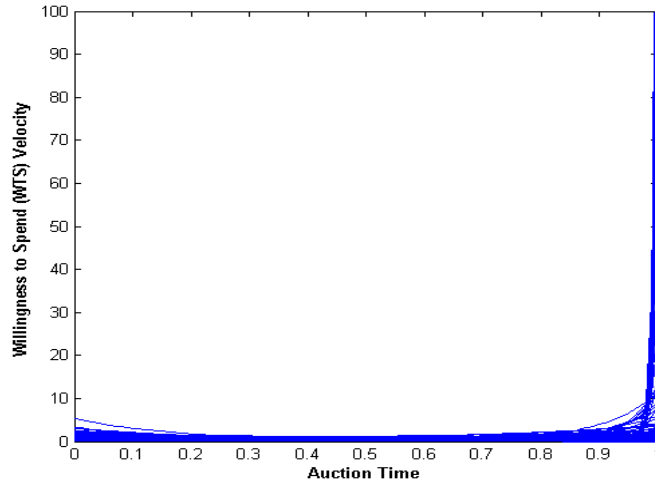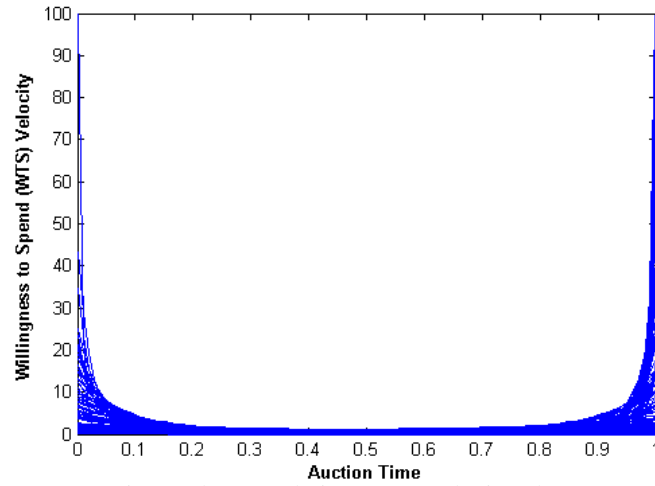
Figure 5a: WTS Velocity Plot

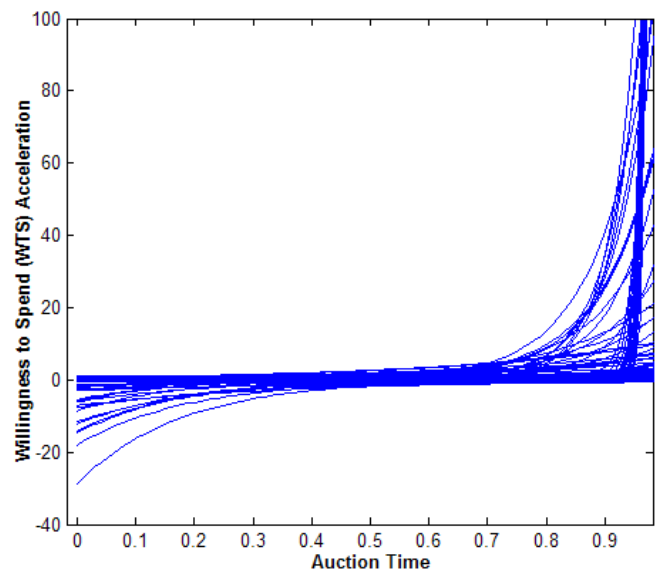Figure 5b: Cumulative WTP Velocity Plot

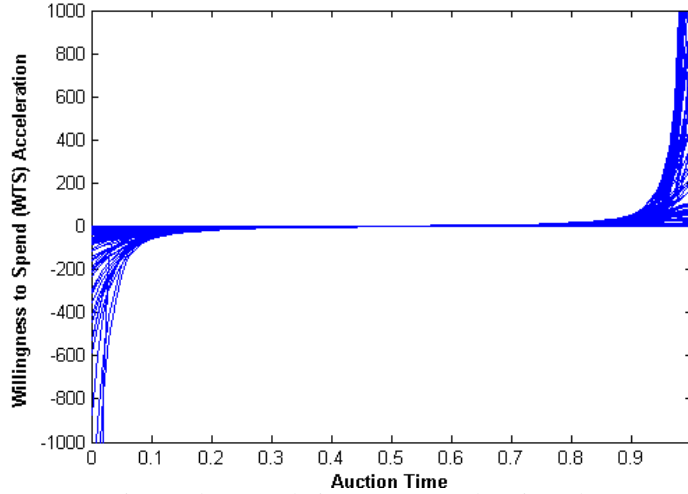Figure 6a: WTS Acceleration Plot

Figure 6b: Cumulative WTP Acceleration Plot

The parameter estimates from the functional regression for each of the five explanatory variables are plotted (Figure 7 to Figure 11) along with confidence bands to indicate their impact on WTS. Results from the functional regression on cumulative WTP are only reported for covariates where there is a discrepancy between WTS and cumulative WTP results. Complete results can be obtained from the auction upon request. For the first bidder behavior characteristics considered, i.e. the current number of bids by the bidder, we find it to have a positive effect on the WTS formation throughout the auction (Figure 7). This indicates that bidders who have a high level of bidding activity will have a high WTS at the beginning of the auction, and will maintain its positive influence on the WTS as the auction progresses. In these auctions, we observe that on average, bidders increase their bidding activity as the auction progresses. Therefore, with most bidders having high level of bidding activity near the end of the auction, its effect on WTS still holds positive. We also found similar results for its effect on cumulative WTP, except that its effect on this dependent variable was higher on WTS. This finding is grounded in the auction theory that suggests that high bidding activity indicates more frequent update of a bidder's private value [Dass, Seymour and Reddy 2010], and thus his/ her WTS (*linkage principle* [Klemperer 2004]) . From the auction house perspective, this observation suggests that auction houses should encourage high activity among the bidders. This can be obtained by strategically selling the items such as sale positioning [Reddy and Dass 2006] and by encouraging competition in auctions [Ariely and Simonson 2003] through inviting specific strong bidders (bidders with high purchase power) interested in particular items [Dass and Reddy 2008].
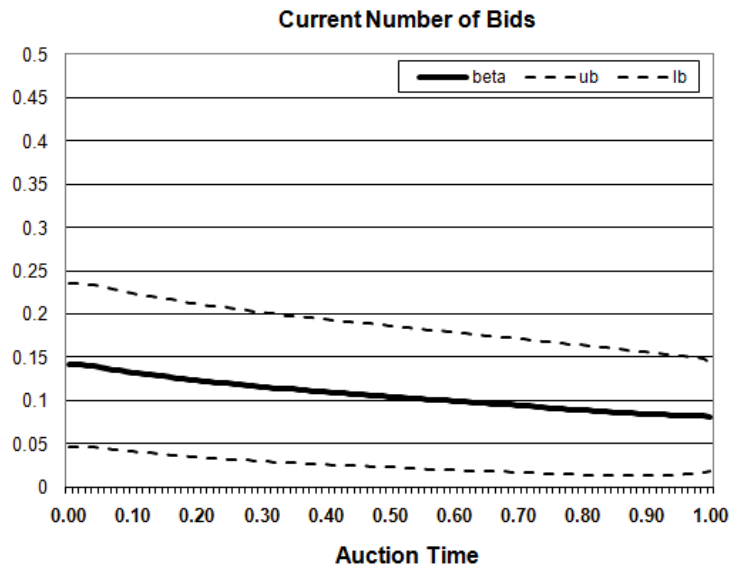

Figure 7: Estimated parameter curves for Current Number of Bids on WTS

Results from the analysis of the effects of other bidder behavior characteristics such as bid time suggest that bid time has a significant positive effect on the WTS dynamics of bidders with the effect being high at the beginning, which gradually reduces near the end (Figure 8a). This indicates that the bidders who bid late in the auction have a high WTS than bidders who bid early. This is exactly in lines with what one would expect in auctions. Therefore, along with adding an insight to the understanding of bidders' WTS, this finding also reverberate the validity of our analysis. Most auction houses heavily market their auction event to make sure that the known bidders participate in the auction. Some even make private calls to specific bidders until the mid-point of the auction event is reached [Dass and Reddy 2008]. Positive and significant effect of late bidders on WTS suggests that auction house managers should engage in such heavy marketing activities until the very end of the auction to encourage more late bidders to participate. Interestingly, we find insignificant effect of bid time on cumulative WTP (Figure 8b) at the beginning of the auction, but become significant as the auction progresses. This suggests that WTS is a sensitive measure in comparison to cumulative WTP, a desired characteristics for a useful bidder measure for the auction houses. We find no significant effect of the bidders' number of proxy bids on both WTS and cumulative WTP dynamics, contrary to what we expected earlier (Figure 9). This finding may be because most bidders prefer to bid personally and not use the automated bidding system provided by the auction houses, thus having no effect on WTS.
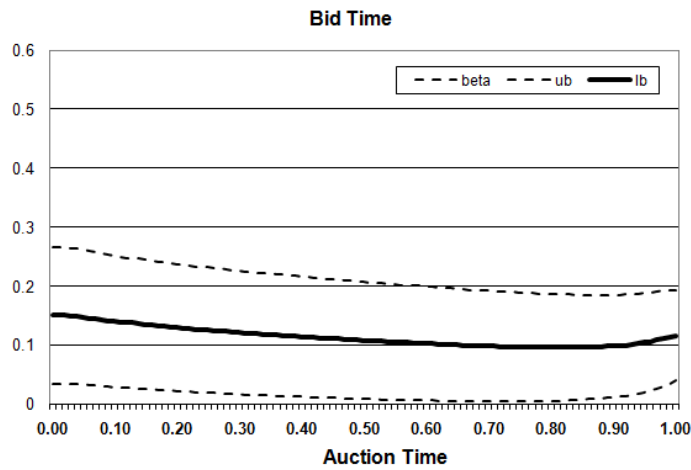

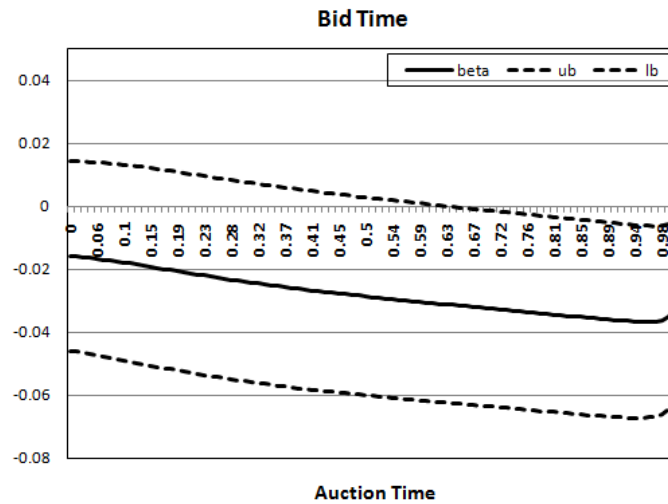Figure 8a: Estimated parameter curves for Bid Time on WTS

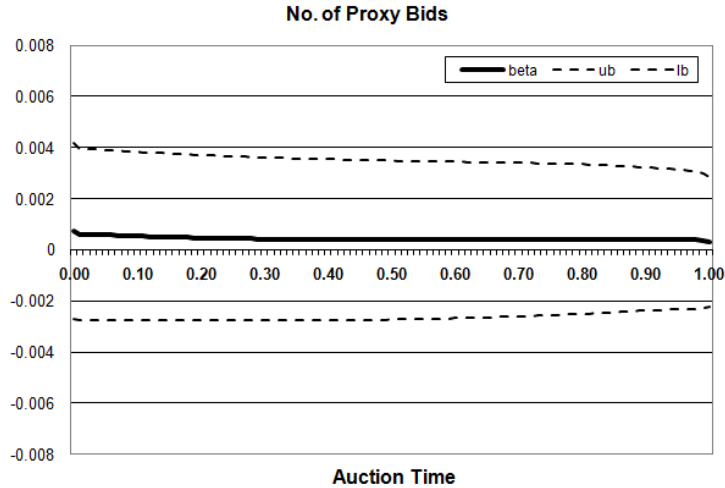Figure 8b: Estimated parameter curves for Bid Time on Cumulative WTP

Figure 9: Estimated parameter curves for Number of Proxy Bids on WTS

Finally, we found both the current number of lots winning and the pre-auction low estimate of lots where the bidder is currently the highest bidder to have no effect[5] on his/ her WTS (and cumulative WTP) as evident by the estimation plot shown in Figure 10 and 11 respectively. This finding relates back to the fact that the context of this study is private value auctions where bidders get value cues mostly from other bidders. Although pre-auction estimates provided by the experts from the auction house signal item values, they are not strong enough to affect WTS. This suggests that the priority of the auction houses should be on encouraging and attracting more bidders in the auction event than spending resource on providing accurate item valuation. Moreover, auction house managers should spend more time and resource to provide an attractive item line-up for the auction event to get more bidders to participate. Moreover, effects of current number of lots winning by bidders on their WTS suggests that auction houses should encourage as many bidders as possible to participate in these auctions in order to increase competition, which is similar to the finding in the extant literature [Reddy and Dass 2006].
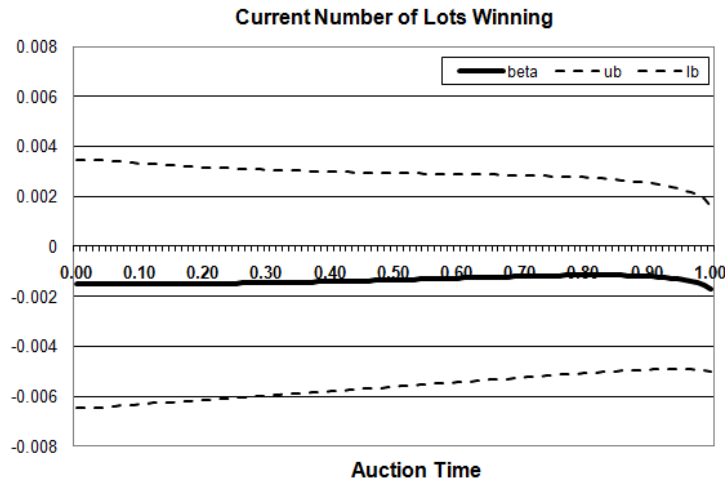

Figure 10: Estimated parameter curves for Current Number of Lot Winning on WTS

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[5] The confidence band includes the zero; hence the effect is not significant.

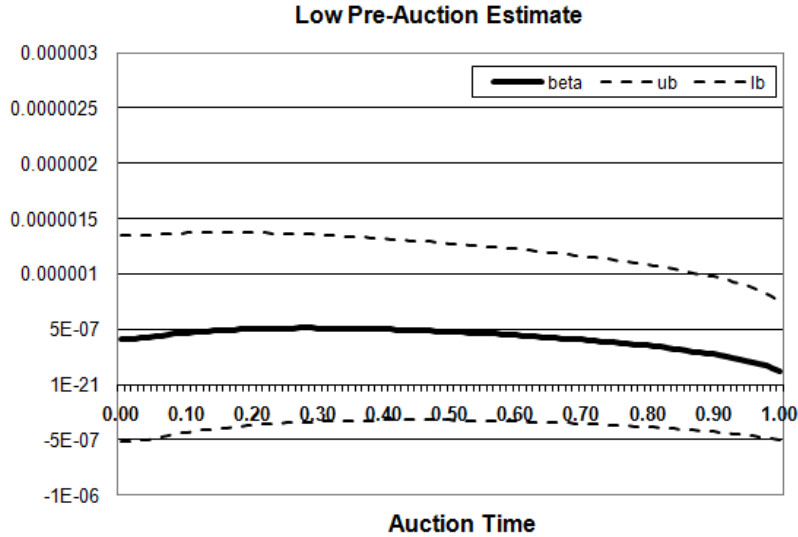**Low Pre-Auction Estimate**

Figure 11: Estimated parameter curves for Pre-Auction Low Estimate of Lots Currently Winning on WTS

6. DISUSSION AND CONCLUSION

In this paper, we investigate *Willingness to Spend* (WTS) dynamics of bidders in a simultaneous online art auction and compare it with cumulative *Willingness to Pay* (WTP) dynamics. Using Functional Data Analysis, we first discover the WTS and cumulative WTP function of participating bidders, and then using functional regression, we explore the effects of three bidder behavior characteristics including current number of bids, bid time, and number of proxy bids and two bidder preference characteristics including current number of lots winning and pre-auction low estimate of the lots they are currently winning, to provide insights on their relationship with the WTS/ cumulative WTP dynamics of bidders during the auction. Our analysis suggests four broad bidding strategies in these online auctions. We also found positive significant effect of current number of bids and bid time and no significant effect of current number of lots winning, pre-auction low estimate of these lots, and number of proxy bids on the WTS dynamics. Analysis of cumulative WTP also suggested similar results with the exception of the effect of bid time, which becomes significant at the end of the auction.

6.1. Implications for Practice

In this paper, we use bidding data from an online simultaneous auction of fine art. With art market becoming popular as an alternative investment source in recent years, importance of online auctions as an art distribution outlet has reached a new level. With auction houses spending heavily prior to the auction to influence bidders' WTP for auctioned items, our work is timely and provides great insights to the managers. For example, our study suggests that managers to spend more resource in attracting bidders who are active while they participate in auctions. Such heavy participation will lead to higher WTS, and higher price levels for the auctioned items. Prior bid history of bidders can be mined to identify these bidders and encourage them to participate in future auctions. We also found that bid time to have a positive effect on bidders' WTS. This suggests that auction house managers also have an opportunity to contact bidders during the auction and encourage them to bid before the auction ends. Although such strategies are currently being used by Artnet.com and SaffronArt.com to increase bidder traffic, our study reconfirms the importance of it.

6.2. Implications for Research

Current auction literature on WTP is focused on common value auctions of single items where the budget constraint plays a significant role in bidders' valuation. This paper extends this literature by investigating bidders' WTS in private value simultaneous online auctions where bidders bid on multiple auctions concurrently and where inter-bidder influence is prominent. In particular, this paper provide insight on how the relative valuation of the items in the bidders' consideration set changes and how it is affected by the auction related factors during the auction process. Furthermore, this paper uses an innovative statistical approach called Functional Data Analysis that presents a new statistical power in analyzing dynamic bidder characteristics in online auctions such as WTS. Particularly, it broadens the scope of many interesting challenges in simultaneous auctions that are not explored earlier. For example, how do items auctioned concurrently compete against each other? Are they better off being competitive in terms of having common bidders fighting for them, or is it better for them being non-competitive? In other words, what is an optimal item line-up for simultaneous auctions such as the one examined in this paper?

This study is an initial attempt to conceptualize and analyze bidders' *Willingness to Spend* in the context of online auctions. While we believe that the unique data and the novel analytical approach provides insights to this important bidder construct, it is not without limitations that could be addressed in future research. First, auction data from one simultaneous online fine art auction is used in the analysis. Since fine art objects are heterogeneous in nature, it will be interesting to see if the results hold if the study is replicated with a different dataset. Second, the sample represents a very specific type of product, with a very high price tag. It will be interesting to see how the results will vary with auction data of low price items. Third, we defined WTS as the total amount a bidder is required to pay at a given time period during the auction if he/ she wins all the items at the current highest bid amount where he/ she is currently the highest bidder. There are possibly many other ways this construct can be measured. Although we limit our study to comparison of WTS and cumulative WTP, future studies are necessary to look at alternative measures of WTS and to compare the effects of the examined factors on them. Fourth, this paper assumes that information flow on the value of an object is uniformly distributed across the auction time. This is a very strong assumption as 1) information distribution of a given item is unknown, and 2) its distribution may vary from item to item. Although such an assumption was necessary to evaluate all items simultaneously, future studies are necessary to explore this issue further.

Rich and detailed bidding data from online auctions provide a fertile ground to investigate various bidding strategies and bidder phenomena in auctions. We believe that our paper takes the first step towards taking advantage of this opportunity. We hope that our work will encourage others to pursue future studies to advance our understanding of online auctions.

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