CONSUMER SHOWROOMING, THE SUNK COST EFFECT AND ONLINE-OFFLINE COMPETITION

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ABSTRACT

This paper studies price competition between the online and offline channels under the effects of showrooming and the sunk cost effect. Consumers who are uncertain about their product valuation might examine the product in a physical store but then switch to buying from the online store at a lower price (i.e., showrooming). We consider the sunk cost effect in a setup involving two competing stores -- online vs. physical -- and consumers that have valuation uncertainty and heterogeneous preferences about visiting the physical store. Our results suggest that the online store may be better off if it targets only one type of consumer -- either direct buyers or switchers, but not both. However, the online store can only do so under strict conditions, so it is more likely to engage in fierce price competition with the physical store to pursue switchers. We also find that high transportation cost may benefit both stores in most circumstances, but more so for the physical store because of the sunk cost effect. Higher sunk cost effect allows both stores to charge higher prices in certain circumstances. In sum, while showrooming is more likely to aggravate the competition, the sunk cost effect might mitigate the competition, benefiting both stores.

Keywords: Online-offline competition; Showrooming; Valuation uncertainty; Sunk cost effect; Pricing game

1. Introduction

When consumers have access to both online and brick-and-mortar (BM) stores, "showrooming" happens: consumers examine a product in the physical store but switch to buying it online [Zimmerman 2012]. Although the online channel offers convenience and probably a lower price, consumers might still value the option of examining the actual products before making their purchases [Levin et al. 2003]. Online technology is still limited in demonstrating product attributes that involve touch, smell, or fit -- attributes that are essential for selling certain products, such as perfume or shoes. Consumers might have difficulty evaluating these products through the online store alone and thus be discouraged from online shopping. For example, in a survey of shoppers, Kacen et al. [2013]

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found that one of the major concerns of online shopping is the uncertainty about getting the right item. Kim and Krishnan [2015] demonstrated that consumers are reluctant to buy products over $50 online if there is a high degree of valuation uncertainty, even when they have accumulated much online shopping experiences. Zhou et al. [2007] concluded that increased online shopping experience does not lessen consumers' perceived risk, while Dai et al. [2014] confirmed that limited physical access to products and sales personnel magnifies the level of perceived risk. To resolve the uncertainty on product valuation, consumers can visit a physical store, where they can see, touch, feel, smell, and try on the product in person and go back to purchase online for lower prices. A 2012 survey of U.S. online shoppers by Click IQ finds that 45.9% of respondents reported showrooming behavior [Balakrishnan et al. 2014]. A report by Codex reveals that 24% percent of online book purchasers checked out the book first in a BM store.

While showrooming seems to be prevalent, there is evidence that trips to BM stores may have more implications than being a showroom. For instance, Farag et al. [2007] found that online searching of product information leads to more trips to BM stores, which do not necessarily convert to online purchases. Forman et al. [2009] showed that transportation cost matters to consumer decisions and offline stores entry in an area reduces consumers' sensitivity to price discount online, while the breadth of the product line at a local store does not matter to consumers. It seems that we cannot readily assume that consumers would certainly free-ride the physical store as a showroom and buy from the online store, given a lower price and no disutility from online shopping.

We propose that consumers are subject to the sunk cost effect. The sunk cost effect is a maladaptive economic behavior that is manifested in a greater tendency to continue an endeavor after an investment in money, effort, or time has been made, due to psychological biases and cognitive limitations. Research has consistently demonstrated the sunk cost effect in human decision making. Arkes and Blumer [1985] conducted experiments in a variety of situations and confirmed the sunk cost effect. For instance, the majority of their subjects who had bought an expensive ticket for a less enjoyable ski trip continued with this trip rather than buying another less expensive ticket for a more enjoyable trip. Similarly, 85% of the subjects decided they would finance a project if they had previously invested in it, while only 17% of the subjects who had made no prior investment would do so. In the context of consumer behavior, Dick and Lord [1998] showed that, after paying a membership fee, consumers are reluctant to switch stores, even if consumer utilities for the chosen and un-chosen stores become equal. Moreover, the sunk cost effect is robust and not easily attenuated. Stanovich and West [2008] attested that the sunk cost effect is not alleviated as cognitive capability increases because similar patterns were found in subjects scoring both high and low on the Scholastic Assessment Test (SAT). Arkes and Blumer [1985] found that giving direct instructions on the sunk cost concept had no significant effect on decision makers' susceptibility to the effect. People might be well aware of the principles of the sunk cost effect, but they tend to ignore, reject, or forget the principles when they make actual decisions [Simonson and Nye 1992; Tan and Yates 1995]. Although the sunk cost effect has been well studied empirically and experimentally, it has been rarely introduced to the analytical research on supply chain management or marketing. With analytical models, Rajagopalan et al. [2015] found that a monopolist service provider should adopt the time-based pricing scheme if the sunk cost bias is small and it should adopt the fixed fee scheme otherwise; and in a competitive setting, a time-based scheme is more likely in markets.

How would showrooming with the sunk cost effect affect the competition between online and offline stores? Some studies have examined firm strategies under the effect of showrooming [Mehra et al. 2013; Balakrishnan et al. 2014]. Mehra et al. [2013] identified three strategies to counter showrooming for the physical store: price matching with the online store, making product matching harder (e.g., creating a possibility that the best-fit product might not be available online), and charging customers for showrooming. Balakrishnan et al. [2014] showed that the ratio between the costs of shopping online and shopping offline determines the equilibrium and hence the existence of showrooming, given heterogeneous consumers and different online shopping return policies. The general conclusion is that showrooming intensifies the price competition between online and offline stores.

By taking the sunk cost effect into account, we propose that consumers' behavior might actually be different from what is described in the literature: some consumers might be reluctant to switch to the online store after a visit to the physical store. For these consumers, visiting the physical store incurs a transportation cost, including the direct cost of travel and the opportunity cost of time and effort. Upon arrival at the store, the transportation cost becomes sunk cost. Carrying the sunk cost effect forward, consumers visiting the physical store have a greater tendency to stick to the channel that has already cost them time, effort, or money, instead of switching. Therefore, we regard the sunk cost effect as a factor that might differentiate consumers' showrooming behavior. The current study incorporates the sunk cost effect into consumers' channel choice and examines the implications for the online-offline competition. Anticipating differentiation in consumers' showrooming behavior, online and offline stores need to apply certain strategies to maximize their profit. Specifically, we investigate the price competition, derive optimal pricing strategies of online and physical stores, and clarify the conditions under which each store can benefit.

We consider a model involving an online store, a physical store, and heterogeneous consumers with valuation uncertainty, following the general assumptions in the literature. The sunk cost effect is modeled as extra utility consumers gain from buying from the physical store, which increases as the transportation cost increases. Our model reveals some interesting insights. First, the consumer purchase decision depends on the distance between the consumer and the physical store. Consumers who are distant from the physical store directly purchase online. Consumers who are nearby visit the physical store but switch to buy online. Consumers at an intermediate distance visit and are reluctant to switch to an online purchase because of the sunk cost effect. Second, although the demand of the online store comes from both direct buyers and switchers, the online store might be better off by targeting only one type of consumers rather than pursuing both types. If transportation costs are low and the valuation risk and the sunk cost effect are high, the online store can be better off by targeting switchers only; conversely, the online store should target direct buyers. Third, both stores are better off in the scenario where they both "ignore" showrooming and play as if no switchers exist. However, both stores have incentives to take "showrooming" into consideration, which aggravates the price competition and decreases profit. Fourth, our comparative analyses suggest that: a high transportation cost may benefit both stores; a low product uncertainty may aggravate pricing competition and hurt both stores; and the sunk cost effect, which attracts more consumers to visit the physical store to resolve their valuation uncertainty, may allow high prices for both stores.

The rest of the paper is organized as follows. We review the literature and clarify our contributions in Section 2. Then we present the model setup, analyze the results, and discuss managerial implications of the findings in Section 3. We conclude by discussing limitations and potential future research directions in Section 4. All proofs are provided in the Appendix.

2. Literature Review

Our work is related to two literature streams. First, we contribute to the literature on the competition between online and offline channels. Some researchers view the online channel as a strategic tool of the manufacturer to make direct sales [Balasubramanian 1998; Chiang et al. 2003; Yao and Liu2005; Fruchter and Tapiero 2005; Liu and Zhang 2006], and the supply chain players thus use certain pricing strategies to achieve optimal results. For instance, Fruchter and Tapiero [2005] showed that the manufacturer charges the same price across both online and offline channels and the introduction of the online store is a win-win strategy where both the customers and the manufacturer are better off. Liu and Zhang [2006] found that retailers might set personalized pricing to intimidate the manufacturer and discourage it from setting up a direct online channel, even if the pricing is worse for the retailer.

Others have studied the effect of quality, service, risk, searching cost, and other factors on the competition between online and offline retailers. For example, Lal and Sarvary [1999] found that the Internet might decrease price competition even when it reduces both the discriminatory power of information regarding merchandise quality and the search cost for pricing information. Pan et al. [2002] showed that in a price competition between a pure play e-tailer and a bricks-and-clicks retailer, the pure play e-tailer generally has a lower equilibrium price. Chun and Kim [2005] investigated how consumer access to the Internet affects pricing and found that both offline and online prices drop as more consumers have access to the Internet; in addition, online prices tend to be higher than offline prices as more consumers are connected to the Internet.

More recent studies explore more new developments in channel structure. Abhishek et al. [2016] found that agency selling (manufacturers sell through e-tailers for fees), is more efficient than reselling and leads to lower retail prices; however, the e-tailers end up giving control over retail prices to the manufacturer. Herhausen et al. [2015] showed that online–offline channel integration leads to a competitive advantage and channel synergies rather than channel cannibalization. Ofek et al. [2011] demonstrated that when the degree of differentiation between retailers is high, retailers that operate dual channels may opt to increase prices drastically and reduce costly store assistance and gain greater profit than the Bricks-only case. Forman et al. [2009] presented empirical evidence that there is channel substitution between local stores and online purchasing, confirming that the disutility of online purchasing and the transportation costs are comparable between the two channels.

Most of these papers have recognized the difference between online and offline channels from the consumer's perspective. Online shopping generally has the advantage of convenience, as well as higher risks, given that the transaction does not happen on the spot, in person. Uncertainties might arise with the retailer or with any party involved in the supply chain that causes faulty products, delays, or missing of shipments, or with payment settlement problems. The consumer also might regret the purchase when the product is not as she expected. Offline stores do not have such uncertainties, but they do incur a transportation cost -- the actual cost of traveling to the store -- as well as the time and effort of visiting a physical store, the opportunity cost that visit entails, and even the personal distaste for the physical shopping experience. Consumers make purchase decisions by weighing the risks and costs of different channels. However, most of the studies consider the risk of online shopping only at the supply end and neglect consumers' valuation uncertainties.

Second, we contribute to the literature on general multi-channel competition that investigates the effect of consumer valuation risk on firms' strategies. Sellers might provide information services to help consumers solve their valuation problems, make an informed purchase decision, and derive higher utility. However, the seller that provides information service faces the problem of free-riding: consumers might use their service but buy from low-price sellers. Wu et al. [2004] investigated whether a seller should provide such an information service and confirmed that, even if free-riding occurs, a seller should establish itself as an information service provider to profit. Shin [2007] showed analytically that when customers are heterogeneous in terms of their opportunity costs for shopping, free riding benefits not only the free-riding retailer, but also the retailer that provides the service. Gu and Tayi [2016] found that consumers' cross-channel search behavior of pseudo-showrooming or the consumer behavior of inspecting one product at a seller's physical store before buying a related but different product at the same seller's online store may allow a multi-channel seller to achieve better coordination through optimal product placement strategies.

In the context of online and offline stores, we consider features that involve actual personal interaction, such as touch, feel, smell, and fit, and suggest that the online store is naturally less informative than the offline store, We do not consider the case in which consumers might free-ride online reviews and then buy from the offline store. Therefore, our study of showrooming can extend the current understanding of information services to the context of online and offline competition -- a setting that has becoming increasingly common with the development of the Internet.

3. Model Setup and Analysis

3.1. Model Setup

Consider an online store (referred to as "she") and a physical store (referred to as "he") that sell an identical product at prices p_1 and p_2, respectively. Consumers face uncertainty about their private valuation for the product before consumption and they know that β fraction of the consumer population perceives a positive valuation *v* on the product (referred as high-type consumers) and the other $1-\beta$ fraction perceives zero valuation on the product (referred as low-type consumers). That is, if a consumer buys from the online store directly, he faces product risk, which is defined as the probability of the item failing to meet the performance requirements originally intended [Dai et al. 2014]. He or she has probability β to be a high-type consumer, with positive valuation, and has probability $1-\beta$ to be a low-type consumer, with zero valuation. A higher β indicates lower product risk.

Following Balasubramanian [1998], we assume heterogeneous costs of using the physical store and homogenous cost of using the online store. Consumers incur travel costs at a (linear) rate *t* per unit distance when visiting a retailer. These costs can include the opportunity cost of time, the real cost of travel, and the implicit cost of inconvenience. Assume that consumers are uniformly distributed on a linear Hotelling line from 0 to 1, with the physical store located at x=0. The distance refers not only to the actual distance, but also to a consumer's general attitude toward the physical store. A consumer located at *x* has to pay a transportation cost *tx* to visit the physical store, where *t* represents the unit transportation cost [Hotelling 1990]. The market is also served by the online store with no market "location" in the conventional sense. We consider that all consumers have zero transportation cost of buying from the online store, because the cost to sample a product and get a price quote from an online store is only a matter of several mouse clicks [Xu et al. 2011].

The physical store can play the role of a "showroom", where the visitors can inspect the product, verify its fit and features, and resolve the valuation uncertainty. Among the visitors of the physical store, $1-\beta$ fraction turns out to be low-type and β fraction turns out to be high-type. The low-type visitors leave the market without buying anything. The high-type visitors obtain one unit of the product, assuming *v* is high enough such that the high-type consumers have positive surpluses. It is worth to note that there are other ways for consumers to resolve the uncertainty without going to the physical store. For instance, many online stores offer free return services, which can be viewed as a way of showrooming but with little transportation cost. Then the consumers do not need to visit the physical store to resolve their valuation uncertainty. Therefore, our study applies to products that are difficult to evaluate without checking in person and not easily to be returned, e.g. large furniture, fresh produce, etc.

Next, the high-type visitors decide where to buy. They have two options: buy at the physical store or switch to the online store. For them, the transportation cost *tx* becomes sunk cost. The higher the transportation cost, the greater tendency for the consumers to stick to the physical store. Specifically, we assume that, after paying a sunk cost *tx* to the physical store, the willingness to pay for the consumption in the same store increases by λtx, where λ can be viewed as the strength of the sunk cost effect. A higher λ means that consumers are less rational and more significantly influenced by the sunk cost in their decision making. If $\lambda=0$, consumers are perfectly rational and are not affected by the sunk cost at all. Each consumer buys at most one unit of the product. Consumers are risk-neutral and make decisions to maximize their own expected surplus. The number of consumers is normalized to one. Table A.1 summarizes the notations.

We model the willingness to pay as an increasing linear function of the sunk cost effect based on the following justifications. First, the more people invest in a choice, the willingness to pay for the choice increases. Such positive relationship has been manifested in experimental evidence we mention before. For instance, 85% of subjects are willing to continue to invest in a project if they have already invested before, while 17% of those who had no prior investment would do so [Arkes and Blumer 1985]. We use the linear form for the sake of simplicity, but we are sure that most of our results hold when using other function forms such as quadric or exponential functions. Using a different function form may lead to different expression and value of the results, but does not affect the essence of the relationships. We are also confident in the specification as it has been used in the literature, though theoretical papers concerning the sunk cost effect is rare. To our best knowledge, Rajagopalan et al. [2015] investigated the impacts of the sunk cost effect in the context of diagnosis-based services and they assumed similar linear relationship between the sunk cost of diagnosing a computer and the customer willingness to continue repairing it.

3.2. Equilibrium Analysis

The sequence of events, depicted in Figure 1, is as follows.

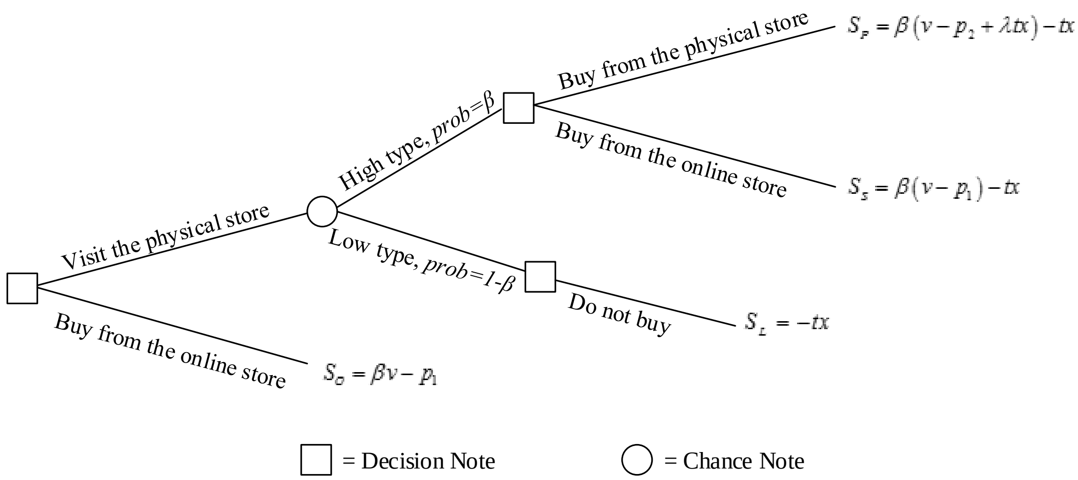


Figure 1: The Sequence of Decisions and Payoffs

In Stage 0, the online store and the physical store set their own retail prices p_1 and p_2 independently and simultaneously.

In Stage 1, given the prices, consumers decide whether to visit the physical store. If a consumer buys from the online store directly, his expected surplus is $\beta v - p_1$. If a consumers located at *x* visits the physical store, her expected surplus is $\beta(v - p_2 + \lambda tx) - tx$ (where the item λtx represents the impact of sunk cost effect) if she continues to buy from the physical store or $\beta(v - p_1) - tx$ if she switches to the online store to buy, whichever is higher. Therefore, consumers located at x buy from the online store directly if x satisfies $\beta v - p_1 > \max\{\beta(v - p_2 + \lambda tx) - tx, \beta(v - p_1) - tx\}$, or equivalently, if $x > \max\{x_0, x_2\}$ where $x_0 = \frac{(1-\beta)p_1}{t}$ and $x_2 = \frac{p_1 - \beta p_2}{t(1-\beta\lambda)}$. Or they visit the physical store when $x < \max\{x_0, x_2\}$. And when $x = \max\{x_0, x_2\}$, consumers at x are indifferent to visiting the physical store or not.

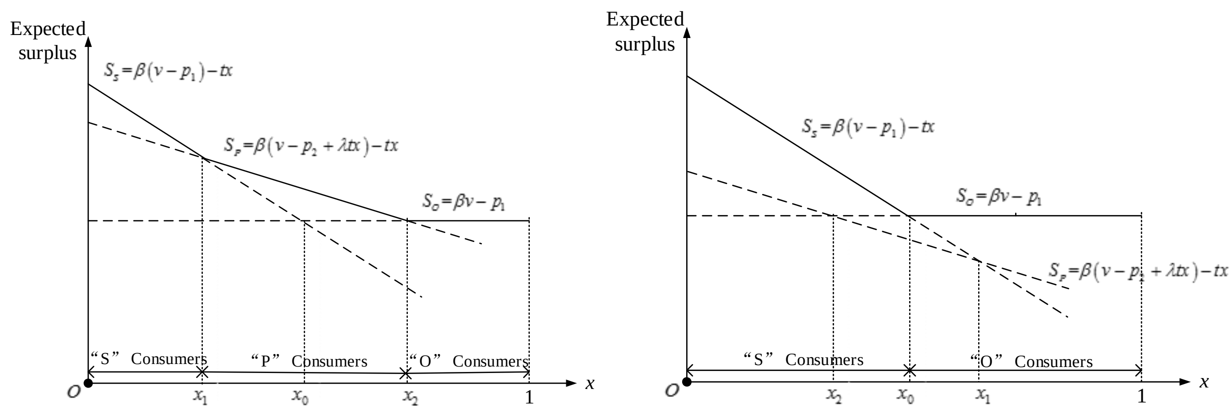
In Stage 2, consumers who visit the physical store recognize their types. The low-type visitors leave the market without buying anything and have zero surpluses, and the high-type visitors decide where to buy. The high-type visitors located at *x* buy from the physical store if $v - p_2 + \lambda tx > v - p_1$ (i.e., if $x > x_1$, where $x_1 = \frac{p_2 - p_1}{t\lambda}$), those located at x switch to buy from the online store if $v-p_2+\lambda tx<v-p_1$ (i.e., $x<x_1$), and those located at x are indifferent from buying from the online store or the physical store when $v-p_2+\lambda tx=v-p_1$ (i.e., $x=x_1$).

Proposition 1 *Given the retail prices p_1 and p_2, consumers' purchasing decisions are described as follows.*

(i) If $p_2<(1+\lambda-\beta\lambda)p_1$ i.e., $x_1<x_0<x_2$, we have the following result. Consumers located at x that satisfies $x>x_2$ buy from the online store directly (referred as "O" consumers); and the others visit the physical store. The low-type visitors leave the market without buying anything. The high-type visitors located at x that satisfies $x_1<x<x_2$ buy from the physical store (referred as "P" consumers); and consumers located at x that satisfies $x<x_1$ switch and buy from the online store (referred as "S" consumers, switchers).

(ii) If $p_2\ge(1+\lambda-\beta\lambda)p_1$ i.e., $x_1\ge x_0\ge x_2$, we have the following result. Consumers located at x that satisfies $x>x_0$ buy from the online store directly (i.e., "O" consumers); and the others visit the physical store. The low-type visitors leave the market without buying anything; and the high-type visitors switch and buy from the online store (i.e., "S" consumers).

The expected surpluses of "O", "P", and "S" consumers are $S_O=\beta v-p_1$, $S_P=\beta\left(v-p_2+\lambda tx\right)-tx$ and $S_S=\beta\left(v-p_1\right)-tx$, respectively (see Figure 2). Each consumer, based on her location x chooses the purchasing behavior that maximizes her surplus as given in Proposition 1.



(a) If $p_2<(1+\lambda-\beta\lambda)p_1$ *(b) If $p_2\ge(1+\lambda-\beta\lambda)p_1$*

Figure 2: Expected Surplus and Consumers' Behavior with Respect to the Location

In the following lemma, we show that neither store would set a price such that the other store has zero market share.

Lemma 1 *Each store prefers including the other store in the market rather than driving the other store out of the market.*

According to Lemma 1, the online store would not drive the physical store out of the market by setting a price that satisfies $p_2\ge(1+\lambda-\beta\lambda)p_1$. This result indicates that Scenario (ii) in Proposition 1 will not occur in equilibrium. Hereafter, we focus on Scenario (i) in Proposition 1.

Facing uncertainty on product valuation, consumers far away from the physical store buy from the online store directly because of high transportation costs. Consumers within a certain distance to the physical store visit the physical store. Among these visitors, the low-type ones leave the market without buying anything, and the high-type ones choose where to buy. Interestingly, consumers near the physical store do not buy from it. For these consumers, the physical store plays the role of "showroom", where they inspect the product but end up buying from the online store. Only the high-type visitors at an intermediate distance to the physical store are reluctant to switch to another channel and buy from the physical store because of the sunk cost effect.

Therefore, the demand of the online store comes from "S" consumers whose amount is βx_1^+ and "O" consumers whose amount is $(1-x_2)^+$ and is given by

$$d_1=\beta x_1^+ + (1-x_2)^+ . \tag{1}$$

The demand of the physical store comes from “P” consumers and is given by

$$d_1 = \beta\left[1-(1-x_2)^+ - x_1^+\right]. \tag{2}$$

The online store and the physical store decide on their own retail prices independently and simultaneously to maximize their own profits given by

$$\pi_1 = (p_1 - w)\left[\beta x_1^+ + (1-x_2)^+\right], \tag{3}$$

$$\pi_2 = (p_2 - w)\left[1-(1-x_2)^+ - x_1^+\right], \tag{4}$$

where w is the wholesale price of the product.

Based on Lemma 1, we focus on the scenario where both stores have positive demand -- that is, where $x_1 < x_2$, $x_1 < 1$ and $x_2 > 0$. Note that “S” consumers exist only if $x_1 > 0$, and “O” consumers exist only if $x_2 < 1$. We focus on $\beta > 1/2$, which means the number of high-type consumers is greater than the number of low-type consumers.

Different equilibrium scenarios can emerge when the stores choose their optimal pricing regimes. The online store has three pricing regimes from which to choose: (i) trying to sell to both “S” consumers and “O” consumers (i.e., setting a retail price satisfying $0 < x_1 < x_2 < 1$); (ii) giving up “O” consumers and targeting “S” consumers (i.e., setting a retail price satisfying $0 < x_1 < 1$ and $x_2 \geq 1$); and (iii) giving up “P” consumers and targeting “O” consumers (i.e., setting a retail price satisfying $x_1 < 0$ and $0 < x_2 < 1$). Correspondingly, the physical store also has three pricing regimes: (i) focusing on the consumers with intermediate distances (i.e., setting a retail price satisfying $0 < x_1 < x_2 < 1$); (ii) attracting all consumers to visit it (i.e., setting a retail price satisfying $0 < x_1 < 1$ and $x_2 \geq 1$); and (iii) trying to sell to all the high-type visitors (i.e., setting a retail price satisfying $x_1 < 0$ and $0 < x_2 < 1$). The following Proposition 2 describes the equilibrium pricing strategy.

Proposition 2 *The equilibrium of the price-setting game can be characterized as follows:*

(i) If $\frac{t}{w} < \min\{U_0, L_1\}$*, the online store sets a price targeting “S” consumers, and the physical store sets a price targeting the rest;*

(ii) If $\min\{U_0, L_1\} \leq \frac{t}{w} < L_1$*, the online store sets a price targeting both “S” and “O” consumers, and the physical store sets a price such that there are no “O” consumers;*

(iii) If $L_1 \leq \frac{t}{w} \leq U_1$*, the online store sets a price targeting both “S” and “O” consumers, and the physical store sets a price targeting “P” consumers;*

(iv) If $U_1 < \frac{t}{w} < L_2$*, the online store sets a price targeting both “S” and “O” consumers, and the physical store sets a price such that there are no “S” consumers;*

(v) If $\frac{t}{w} \geq L_2$*, the online store sets a price targeting “O” consumers, and the physical store sets a price targeting the rest;*

Where $U_0 = \frac{3\bar{\beta}}{3-\beta\lambda-\lambda}$, $L_1 = \frac{\bar{\beta}[(3-\lambda\beta)\gamma_2 - \lambda]}{(\gamma_1-\lambda)[(3+\beta\lambda)\beta\gamma_1 - 2(2\beta-1)\lambda]}$, $U_1 = \frac{\bar{\beta}[(\gamma_1-\lambda)\lambda(4-2\beta+3\beta\gamma_1)\gamma_2 - (4\gamma_2-\beta\gamma_1^2)\sqrt{\lambda\gamma_2}]}{(\gamma_1-\lambda)[(4-4\beta+\beta\gamma_1)\lambda\gamma_2 - (4\gamma_2-\beta\gamma_1^2)\sqrt{\lambda\gamma_2}]}$,

$L_2 = \frac{\bar{\beta}(\gamma_1 + 3\sqrt{\lambda\gamma_2})}{(\gamma_1-\lambda)(4-5\gamma_1+3\sqrt{\lambda\gamma_2})}$, $\bar{\beta} = 1-\beta$, $\gamma_1 = 1+\lambda-\beta\lambda$ *and* $\gamma_2 = \lambda+\beta-\beta^2\lambda$.

For easy reference, we call the first case “Scenario S-P” which means “S” and “P” consumers exist in equilibrium, the second case “Boundary S-P” which means “S” and “P” consumers exist in equilibrium and the physical store sets price on the boundary x_2=1 where no “O” consumers exist, the third case “Scenario S-P-O” which means “S”, “P” and “O” consumers exist in equilibrium, the fourth case “Boundary P-O” which means “P” and “O” consumers exist in equilibrium and the physical store sets price on the boundary x_1=0 where no “S” consumers exist, and the last case “Scenario P-O” which means “P” and “O” consumers exist in equilibrium. The equilibrium prices, demands, and profits of both stores are given in Table 1.

The result shows that under certain conditions, the online store can be better off if it targets only one type of consumer -- either direct buyers (i.e. "O" consumers) or switchers (i.e. "S" consumers), but not both. The intuition is as follows. When the showrooming behavior is potential, the online store and the physical store compete at two ends (as showed by x_1 and x_2 in Figure 2(a)) to expand their own sales. The market and product characteristics may have differentiated effects on the store power at the two ends. For instance, if t is higher, the online store has greater power to compete at x_2 end but smaller power at x_1, but it works oppositely on the physical store. Under certain conditions, either store may choose to compete at only one end where she/he has a greater power, and give up the competition at the other end. This will lead to a less aggressive pricing competition. However, in most cases, both stores have incentives to compete at both ends. It causes fierce pricing competition and hurts both stores.

Specifically, the ratio of the unit transportation cost t to the wholesale price w, the strength of the sunk cost effect λ , and the fraction of high-type consumers β together determine the equilibrium scenarios (see Proposition 1 and Figure 3).

If t and β are extremely low, and λ and w are extremely high, the equilibrium occurs in Scenario S-P. If t and β are relatively low, and λ and w are relatively high, the equilibrium occurs in Boundary S-P. In both Scenario S-P and Boundary S-P, all consumers visit the physical store to reveal their type. High-type visitors who are far from the physical store buy from it, and those near to the physical store switch to buy from the online store. Nevertheless, the pricing regimes in these two scenarios are different. In Boundary S-P, the physical store deliberately sets a retail price to attract all consumers to visit it. The online store uses potential direct buyers as leverage to compete with the physical store. Note that the scenario of Boundary S-P exists only if λ is high. If λ is low, aiming to attract all consumers to visit is not profitable for the physical store because the visitors have a high tendency to switch. In Scenario S-P, the online store has no intention of attracting direct buyers; it focuses on switchers.

If t and β are extremely high, and λ and w are extremely low, the equilibrium occurs in Scenario P-O. If t and β are relatively high, and λ and w are relatively low, the equilibrium occurs in Boundary P-O. All visitors of the physical store are reluctant to switch. We see no "showrooming" behavior. Consumers far from the physical store buy from the online store directly. Consumers near the physical store visit it, and high-type visitors buy from it. Nevertheless, the pricing regimes in these two scenarios are different. In Boundary P-O, the online store leverages potential switchers to compete with the physical store, and the physical store deliberately sets a price to discourage all visitors from switching. In Scenario P-O, both stores behave as if no "showrooming" appears.

If the parameters t, β, λ and w are all in an intermediate range, Scenario S-P-O happens. The online store pursues both direct buyers and switchers. The physical store tries to attract visitors and to discourage switching. The competition is most fierce in this scenario.

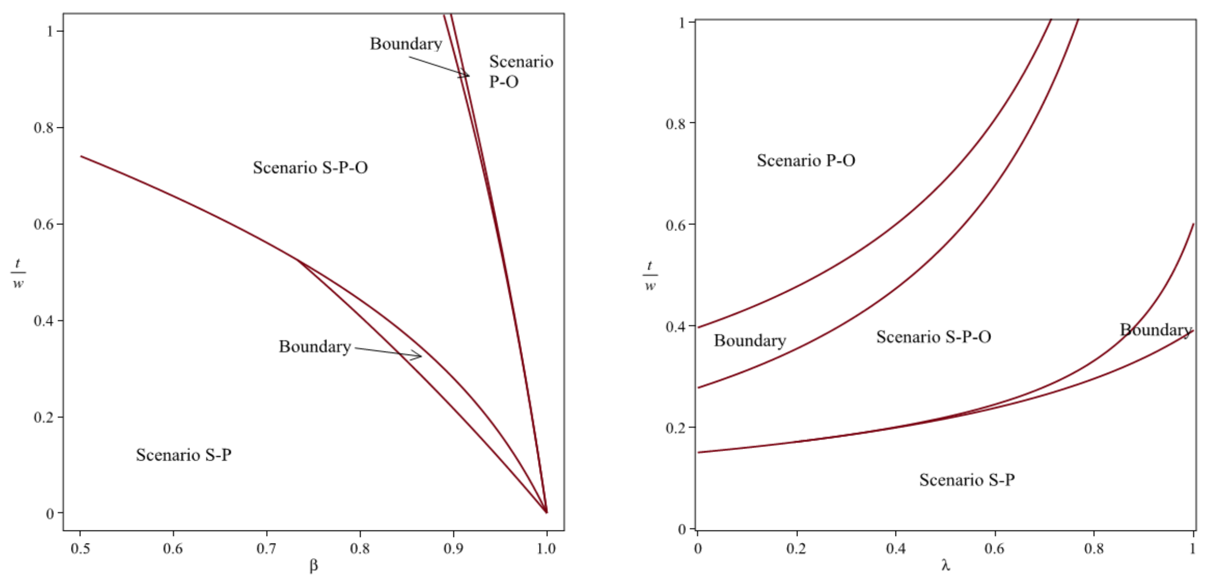
In terms of pricing, we find that in Scenario S-P, Boundary S-P and Scenario S-P-O, where t and β are extremely low and λ and w are extremely high, the price of the online store is lower than that of physical store ($p_1 < p_2$). The online store's low-price strategy aims to attract switchers. In Scenario P-O and Boundary P-O where t and β are high and λ and w are extremely low, the price of the online store is not lower than that of the physical store ($p_1 \geq p_2$). In this situation, there is no switcher. Not surprisingly, the overall price and profit of both firms are significantly higher in this latter situation. When the sunk cost effect makes it possible for no switching, online retailer actually has increasing power in pricing. After both stores become aware of showrooming and try to compete for switchers, both retailers lower their retail prices, the competition gets fiercer, and both are worse off.

Proposition 2 states that in most circumstances, the optimal market strategy of the online retailer is to focus on either direct buyers or switchers, but not both. However, recall that Scenario P-O is viable only when t and β are extremely high, and λ and w are extremely low. Such cases are rather rare. Thus, most likely the online store would be chasing switchers with lower prices, so would the physical store. Such intense price competition may end up driving the one with higher operational costs out of business. This finding may provide a possible explanation for the failure of thousands of physical stores in the US. For them to survive, as our results suggest, the physical stores should try their best to discourage the switching behavior. Thus our perspective may provide an explanation for the recent trend of reinventing the role of physical stores to offer more location-inspired, value-added services to improve consumer experiences.

Table 1: Equilibrium Prices, Demands, and Profits in Each Scenario

Scenario	S-P	Boundary S-P*	S-P-O	Boundary P-O	P-O
p_1	$w+\frac{\lambda t}{3}$	$\frac{\beta w-(\gamma_1-\lambda)t}{\bar{\beta}-\beta}$	$\frac{(\beta\gamma_1+2\gamma_2)w+2\lambda(\gamma_1-\lambda)t}{4\gamma_2-\beta\gamma_1^2}$	$\frac{w+(\gamma_1-\lambda)t}{2-\beta}$	$\frac{(2+\beta)w+2(\gamma_1-\lambda)t}{3}$
p_2	$w+\frac{2\lambda t}{3}$	$\frac{w-2(\gamma_1-\lambda)t}{\bar{\beta}-\beta}$	$\frac{(2+\gamma_1)\gamma_2 w+(\gamma_1-\lambda)\gamma_1\lambda t}{4\gamma_2-\beta\gamma_1^2}$	$\frac{w+(\gamma_1-\lambda)t}{2-\beta}$	$\frac{(1+2\beta)w+(\gamma_1-\lambda)t}{3\beta}$
D_1	$\frac{\beta}{3}$	$\frac{(2-\gamma_1)t-\bar{\beta}w}{(\bar{\beta}-\beta)\lambda t}$	$\frac{\gamma_2[2(\gamma_1-\lambda)t-\bar{\beta}(2-\beta\gamma_1)w]}{(\gamma_1-\lambda)(4\gamma_2-\beta\gamma_1^2)t}$	$\frac{(\gamma_1-\lambda)t-\bar{\beta}w}{(2-\beta)(\gamma_1-\lambda)t}$	$\frac{2}{3}-\frac{\bar{\beta}w}{3(\gamma_1-\lambda)t}$
D_2	$\frac{2\beta}{3}$	$\frac{\beta[\bar{\beta}w-(\gamma_1-\lambda)t]}{(\bar{\beta}-\beta)\lambda t}$	$\frac{\beta[(\gamma_1-\lambda)\gamma_1 t-\bar{\beta}(\gamma_1-2\gamma_2)w]}{(1-\beta\lambda)(4\gamma_2-\beta\gamma_1^2)t}$	$\frac{\beta\bar{\beta}[w+(\gamma_1-\lambda)t]}{(2-\beta)(\gamma_1-\lambda)t}$	$\frac{\beta}{3}+\frac{\beta\bar{\beta}w}{3(\gamma_1-\lambda)t}$
π_1	$\frac{\beta\lambda t}{9}$	$\frac{\beta[\bar{\beta}w-(\gamma_1-\lambda)t]^2}{(\bar{\beta}-\beta)^2\lambda t}$	$\frac{\lambda\gamma_2[2(\gamma_1-\lambda)t-\bar{\beta}(2-\beta\gamma_1)w]^2}{(1-\beta\lambda)(4\gamma_2-\beta\gamma_1^2)^2 t}$	$\frac{[(\gamma_1-\lambda)t-\bar{\beta}w]^2}{(2-\beta)^2(\gamma_1-\lambda)t}$	$\frac{[2(\gamma_1-\lambda)t-\bar{\beta}w]^2}{9(\gamma_1-\lambda)t}$
π_2	$\frac{4\beta\lambda t}{9}$	$\frac{2\beta[\bar{\beta}w-(\gamma_1-\lambda)t][(2-\gamma_1)t-\bar{\beta}w]}{(\bar{\beta}-\beta)^2\lambda t}$	$\frac{\beta\lambda[(\gamma_1-\lambda)\gamma_1 t-\bar{\beta}(\gamma_1-2\gamma_2)w]^2}{(\gamma_1-\lambda)(4\gamma_2-\beta\gamma_1^2)^2 t}$	$\frac{\beta\bar{\beta}[(\gamma_1-\lambda)t-\bar{\beta}w][w+(\gamma_1-\lambda)t]}{(2-\beta)^2(\gamma_1-\lambda)t}$	$\frac{[(\gamma_1-\lambda)t+\bar{\beta}w]^2}{9(\gamma_1-\lambda)t}$

* This equilibrium scenario may occur only if $L_1 > U_0$ i.e. $\lambda > \frac{2+2\beta^2-\beta-4\beta^3}{\beta(1-\beta)(4\beta^2+2\beta+1)}$



(a) Equilibrium scenarios with respect to β and $\frac{t}{w}$ (λ =0.85) (b) Equilibrium scenarios with respect to λ and $\frac{t}{w}$ (β =0.85)

Figure 3: Equilibrium Scenarios

3.3. Comparative Analysis

In this section, we examine how changes in the unit transportation cost t, in the fraction of high-type consumers β, and in the strength of the sunk cost effect λ affect prices and profits in the following propositions.

Proposition 3 *For both stores, the prices and profits increase in t except when the equilibrium occurs in Boundary S-P.*

Proposition 3 suggests, in most scenarios, a high t mitigates the price competition of the online and physical stores, thus resulting in high profits for both stores. On one hand, a high t benefits the online store by discouraging consumers from visiting the physical store and encouraging them to buy directly, even with valuation uncertainty. On the other hand, a high t benefits the physical store because the high sunk cost discourages switchers. Particularly when the equilibrium occurs in Scenario S-P, where all consumers visit the physical store, a high t benefits the physical store more than the online store. However, if the equilibrium falls in Boundary S-P, a high t aggravates price competition and decreases the profit of both firms. In this scenario, the objective of the physical store is to attract all consumers to visit it. A high t discourages consumers from visiting, and the physical store has to decrease its price. Therefore, the competition is aggravated, and both firms are worse off.

Proposition 3 states that higher transportation cost actually alleviates the competition between the online and offline retailers under the influence of the sunk cost effect. It has significant implications for the stores' strategies. "Transportation cost" can be interpreted as any cost, time and effort as well as psychological cost it takes to get to the store and examine the products. It might be feasible to deliberately make it more difficult for consumers to access the product. For example, outlet malls, which offer discounted prices, often take long-distance drives to get to and incur high transportation costs. Now while major department stores (e.g., Sears and Kmart) or brand retailers (e.g., Gymboree and Crocs) are closing hundreds of stores, outlet malls seem to continue to grow. According to our results, it might be that the high transportation costs of going to outlet malls make them to be in less fierce competition with online retailers, which are recognized as the major reason why physical stores are failing. For online retailers, it suggests that product categories that take time, effort or psychological toll to find out about quality could be good business. For instance, online sales of car parts or furniture have been reported to show significant growth.

Proposition 4 (*i) For the online store, the price and the profit decrease in β if the equilibrium occurs in Boundary S-P and if the equilibrium occurs in Scenario S-P-O, Boundary P-O or Scenario P-O and t and λ are high.*

(ii) For the physical store, the price decreases in β if the equilibrium occurs in Boundary S-P, Scenario S-P-O and Scenario P-O and if the equilibrium occurs in Boundary P-O and λ is high; and the profit decreases in β if the equilibrium occurs in Scenario P-O and if the equilibrium occurs in Boundary S-P or Scenario S-P-O and λ is low and if the equilibrium occurs in Boundary P-O and λ is high.

For the online store, a higher β means a lower risk of valuation uncertainty and thus higher surplus for direct buyers. Meanwhile, for the physical store, it means more potential buyers. It seems that both stores may benefit from a market of more high-type consumers. However, under the influence of the sunk cost effect, a high β aggravates the pricing competition and hurt both firms if (i) λ is high, i.e. strong sunk cost effect and (ii) the equilibrium occurs in scenarios where there are potential direct buyers, as Proposition 4 suggests.

The intuition is as follows. The online store and the physical store compete in two aspects. First, the online store needs to encourage direct buyers, while the physical store needs to encourage consumers to visit. Second, they compete for visitors to the physical store by encouraging switching or onsite buying. A higher β increases the online store's pricing power in the first aspect. If λ is high, i.e. consumers are highly irrational and strongly affected by the sunk cost, the online store has disadvantages in the competition for switchers and thus would focus more on the competition in the first aspect. Since direct buyers are less willing to pay than switchers, the online store has to lower the price to encourage direct buyers. The lower price at the online store will decrease the number of visitors to the physical store, which leads to the physical store to reduce price too. Thus, the stores are again engaged in price competition. To sum, in a market of more high-type consumers, the sunk cost effect may increase the competition between the online and physical stores.

Proposition 5 *(i) For the online store, the price and the profit increase in λ if the equilibrium occurs in Scenario S-P or Boundary S-P and if the equilibrium occurs in Scenario S-P-O and λ and β are low.*

(ii) For the physical store, the price and the profit of decrease in λ if the equilibrium occurs in Boundary P-O or Scenario P-O and if the equilibrium occurs in Scenario S-P-O and λ and β are high.

Proposition 5 reveals that when the sunk cost effect is at the low level, both stores may benefit when there are switchers. The sunk cost effect works both directly and indirectly on the price competition. The direct effect is that a higher λ increases the visitors' reluctance to switch. This effect benefits the physical store and hurts the online store. The indirect effect is that a higher λ induces more consumers to visit the physical store to resolve their valuation uncertainty. Having more consumers who have already resolved the valuation uncertainty, the online store can charge a higher price. In Scenario S-P or Boundary S-P, there are no direct buyers, the indirect effect is significant for the online store and may dominate the direct effect for a low λ. Similarly, in Scenario S-P-O, if β is low, consumers would not directly buy online except for very low price. So it is better for the online store to focus more on switchers and to encourage them to visit the physical stores, as in Scenario S-P or Boundary S-P. Therefore, a higher λ benefits both stores in these situations.

When there are no switchers (Boundary P-O or Scenario P-O), a higher λ aggravates price competition, and leads to lower profits for both stores. Because the pricing restrains switching, the physical store cannot leverage the sunk cost effect. Meanwhile, the online store has no market for switchers, so it has to drop the price low enough to encourage consumers to buy from it directly.

It seems that the sunk cost effect not only works for the physical store, but sometimes benefits the online store as well if it mainly targets the switchers. A higher λ increases the pool of potential switchers, as more consumers would visit the physical store because of the higher surplus of buying onsite. With updated valuation certainty the online store is able to charge a higher price. So the sunk cost effect can increase the online store's pricing power and achieve higher profit.

4. Conclusion

This paper studies the price competition between the online and offline channels under the effects of showrooming and the sunk cost effect. We consider a setup with two competing stores, online vs. physical; consumers who have valuation uncertainty and heterogeneous preferences for visiting the physical store; and the sunk cost effect. We follow the literature on showrooming but introduce the sunk cost effect as a factor in consumers' decision about the channel that they buy from. We depict the consumer choice pattern, derive different equilibrium conditions, and investigate how prices and profits change for both channels when the transportation cost and the sunk cost effect change.

Our analysis suggests that showrooming might aggravate the price competition. In terms of firm strategies, both stores would be better off if they "ignore" showrooming and set up prices as if it were not there. However, both online and physical stores have incentives to deviate and to engage in price competition, although it results in a lower profit. The online store might be better off if it targets only one type of consumers. If transportation cost and the sunk cost effect are high enough, the online store is better off targeting only switchers. The rationale is that when more consumers visit the physical store because of the high surplus from the sunk cost effect, more consumers are certain about their valuation. If the market is large enough, the online store can leverage this resolution of valuation risk to charge a higher price. When conditions are reversed, the online store should target direct buyers only.

Interestingly, the sunk cost effect might also mitigate the price competition and benefit both online and offline stores. First, both stores anticipate fewer switchers. Second, a high sunk cost effect attracts more consumers to the physical store, by increasing the surplus of visits. The physical store benefits from fewer switchers, while the online store benefits from an increased number of buyers with valuation certainty, which can support a higher price. With similar reasoning, high transportation costs also might mitigate the price competition. In addition, lower valuation uncertainty aggravates price competition because the physical store has to lower prices to attract consumers to visit. Such a tendency actually has been seen in the current practice of e-commerce and offline competitors.

We acknowledge several limitations of our study. First, we model the heterogeneity of consumer valuation with a simple Bernoulli distribution by assuming that a fraction of consumers are high-type consumers and the others are low-type consumers. More general distributions -- for example, uniform distribution and normal distribution -- could be considered in future research. Second, we did not model the disutility of online shopping. Evidence shows that most products are less acceptable from an online store than a physical store and that consumers perceive a lower valuation from online shopping than from offline shopping. Such disutility should play a role after the consumer visits the physical store. It may decrease the consumer's willingness to switch, while the sunk cost effect increases the consumer's willingness to buy from the physical store. Future research should consider possible interesting interactions between the two effects. Third, in modeling the impact of the sunk cost effect on consumer surplus, we assume for simplicity that the degree of reluctance is linear with the strength of the sunk cost effect. Future research should consider that the strength of the sunk cost effect might not be constant, but increase with the investment.

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Appendix

Table A.1: Summary of Notation

Notation	***Definition***
p_1	The price of the online store
p_2	The price of the physical store
β	The fraction of the high-type consumers
v	The perceived valuation of the high-type consumers
x	The consumer's distance to the physical store
t	The unit transportation cost
λ	The strength of the sunk cost effect
d_1	The demand of the online store
d_2	The demand of the physical store
w	The wholesale price of the product
π_1	The profit of the online store
π_2	The profit of the physical store

Proof of Proposition 1

Recall that the surpluses of "O", "P", and "S" consumers are $S_O = \beta v - p_1$, $S_P = \beta\left(v - p_2 + \lambda tx\right) - tx$ and $S_S = \beta\left(v - p_1\right) - tx$, respectively. Let x_0 denote the location value at which a consumer is indifferent between buying from the online store directly ("O" behavior) and buying from the online store after visiting the physical store ("S" behavior). Let x_1 denote the location value at which a consumer is indifferent between visiting and buying from the physical store ("P" behavior) and buying from the online store after visiting the physical store ("S" behavior). Let x_2 denote the location value at which a consumer is indifferent between visiting and buying from the physical store ("P" behavior) and buying from the online store directly ("O" behavior). We have $x_0 = \frac{(1-\beta)p_1}{t}$, $x_1 = \frac{p_2 - p_1}{\lambda t}$ and $x_2 = \frac{p_1 - \beta p_2}{t\left(1-\beta\lambda\right)}$.

Because $\frac{\partial S_{VO}}{\partial x} = -t < \frac{\partial S_{VP}}{\partial x} = -(1-\lambda)t < \frac{\partial S_{NO}}{\partial x} = 0$, for any given prices p_1 and p_2, "O" consumers must have higher x than any "P" consumers, who in turn must have higher x than any "S" consumers. The "P" consumers exist if and only if $x_1 < x_2$ i.e. $p_2 < \left(1+\lambda-\beta\lambda\right)p_1$. Thus, we can characterize the consumer types by partitioning the range of possible x values into three intervals (some possibly empty) corresponding, from low to high values, to "O" consumers, "P" consumers, and "S" consumers, respectively. Figure 2, showing representative surplus functions for given prices, illustrates this structure.

(i) If $p_2 < \left(1+\lambda-\beta\lambda\right)p_1$ i.e. $x_1 < x_0 < x_2$, then we see the surplus functions with respect to the location as given in Figure 2(a). Consumers with x ranging from 0 to x_1 are "S" consumers, those with x between x_1 and x_2 are "P" consumers, and consumers with x greater than x_2 are "O" consumers.

(ii) If $p_2 \ge \left(1+\lambda-\beta\lambda\right)p_1$ i.e. $x_1 \ge x_0 \ge x_2$, then we see the surplus functions with respect to the location as given in Figure 2(b). Consumers with x ranging from 0 to x_0 are "S" consumers and those with x greater than x_0 are "O" consumers.

Proof of Lemma 1

We first establish the result for the online store. If the physical store lowers its price to the wholesale price and still has no sales, it quits the market. To price the physical store out of the market, the online store either (i) sets a price satisfying $x_1 \ge x_2$ i.e. $p_1 \le \frac{p_2}{1+\lambda-\beta\lambda} = \frac{w}{1+\lambda-\beta\lambda} < w$, (ii) sets a price on the boundary $x_2 = 0$ i.e. $p_1 = \beta p_2 = \beta w < w$ or (iii) sets a price on the boundary $x_1 = 1$ i.e. $p_1 = p_2 - \lambda t = w - \lambda t < w$. These prices are lower than the wholesale price, i.e. $p_1 < w$. Hence, the online store is reluctant to price the physical store out of market.

We next establish the result for the physical store. As with the physical store, if the online store lowers its price to the wholesale price and still has no sales, it quits the market. To price the online store out of the market, the physical

store needs to set its price satisfying $x_1 \le 0$ and $x_2 \ge 1$ i.e. $p_2 \le \min\left\{p_1, \frac{p_1 - t(1-\beta\lambda)}{\beta}\right\} = \min\left\{w, \frac{w-(1-\beta\lambda)t}{\beta}\right\} \le w$.
Thus, the physical store cannot drive the online store out of the market by setting a price higher than the wholesale price.

Proof of Proposition 2

Figure A.1 depicts the feasible regions of each scenario with respect to p_1 and p_2. According to Lemma 1, the regions where the physical store has no sales (i.e., $p_2 \ge (1+\lambda-\beta\lambda)p_1$) and where the online store has no sales (i.e., $p_2 \le \min\left\{p_1, \frac{p_1 - t(1-\beta\lambda)}{\beta}\right\}$) are not feasible. The feasible regions include Scenario P-O, Scenario S-P-O, and Scenario S-P, as well as the boundaries separating Scenario S-P-O and Scenario P-O (referred to as Boundary P-O) and separating Scenario S-P-O and Scenario S-P (referred as Boundary S-P).

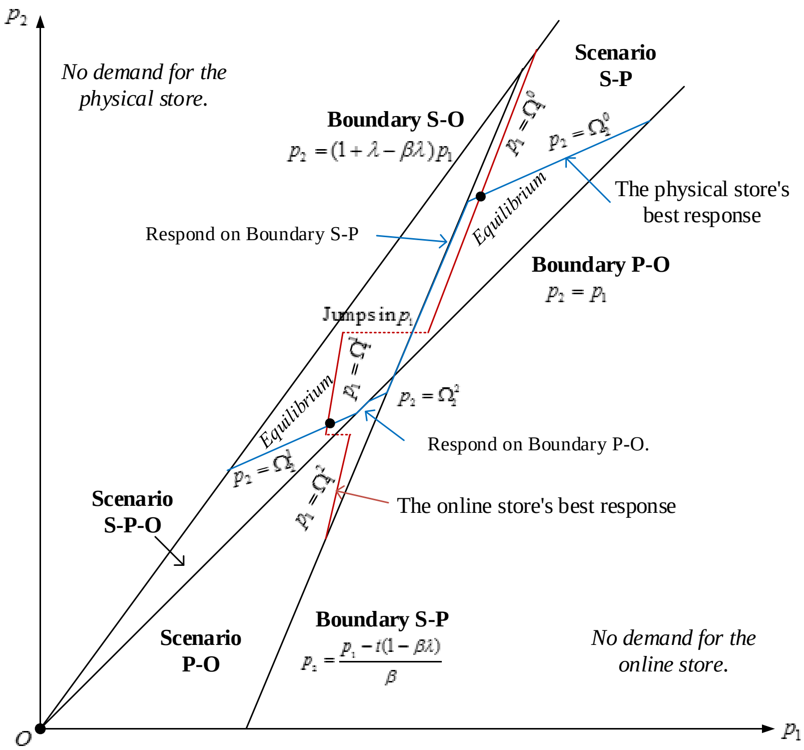

Figure A.1: Feasible Regions and Best Responses

- **Demand and best response functions in interior of each scenario**

We derive each firm's demand and best response functions in the interior of each scenario, assuming that the validity conditions for the scenario holds. Let Π_1^S and Π_2^S denote the profit functions of the online store and the physical store in inner Scenario *S*, respectively. Note that the superscript *S* = 0, 1, or 2 represents Scenario S-P, Scenario S-P-O, or Scenario P-O, respectively. Let Ω_i^S denote store *i*'s best response in the inner of Scenario S, where *i* = 1 or 2 represents the online store and the physical store, respectively, and *S* = 0, 1, or 2. For every scenario, we derive each store's best response function within the interior of the scenario by maximizing the profit based on the scenario's interior demand function, assuming that the scenario validity conditions and bounds on demand (i.e., without explicitly imposing these conditions). The interior best responses are concluded in Table A.2.

Table A.2: The Best Responses and Profits in the Interior of Each Scenario

Scenario	Store	Inner best response Ω_i^S	Profit $\prod_i^S$
S-P	Online store	$\Omega_1^0 = \frac{p_2 + w}{2}$	$\Pi_1^0 = \frac{\beta(p_2 - w)^2}{4\lambda t}$
	Physical store	$\Omega_2^0 = \frac{p_1 + w + \lambda t}{2}$	$\Pi_2^0 = \frac{\beta(p_1 - w + \lambda t)^2}{4\lambda t}$
S-P-O	Online store	$\Omega_1^1 = \frac{\beta\gamma_1 p_2 + \gamma_2 w + (1-\beta)\lambda t}{2\gamma_2}$	$\Pi_1^1 = \frac{[\beta\gamma_1 p_2 - \gamma_2 w + (1-\beta)\lambda t]^2}{4\gamma_2(1-\beta\lambda)\lambda t}$
	Physical store	$\Omega_2^1 = \frac{\gamma_1 p_1 + w}{2}$	$\Pi_2^1 = \frac{\beta(\gamma_1 p_1 - w)^2}{4(1-\beta\lambda)\lambda t}$
P-O	Online store	$\Omega_1^2 = \frac{\beta p_2 + w + t(1-\beta\lambda)}{2}$	$\Pi_1^2 = \frac{[\beta p_2 - w + (1-\beta\lambda)t]^2}{4(1-\beta\lambda)t}$
	Physical store	$\Omega_2^2 = \frac{p_1 + \beta w}{2\beta}$	$\Pi_2^2 = \frac{(p_1 - \beta w)^2}{4(1-\beta\lambda)t}$

Scenario S-P ($0 < x_1 < 1$ ***and*** $x_2 \geq 1$***)***

In this scenario, the profit of the online store is

$$\prod_1^0 = \frac{\beta(p_2 - p_1)}{t\lambda}(p_1 - w)\,. \tag{A. 1}$$

The profit function is concave in p_1. The interior best response of the online store can be got by solving $\partial\prod_1^0/\partial p_1 = 0$ i.e.

$$\Omega_1^0 = \frac{p_2 + w}{2}\,. \tag{A. 2}$$

In this scenario, the profit of the physical store is

$$\prod_2^0 = \beta(1 - \frac{p_2 - p_1}{t\lambda})(p_2 - w)\,. \tag{A. 3}$$

The profit function is concave in p_2. The interior best response of the physical store can be got by setting $\partial\prod_2^1/\partial p_2 = 0$ i.e.

$$\Omega_2^0 = \frac{p_1 + w + \lambda t}{2}\,. \tag{A. 4}$$

Scenario S-P-O ($0 \leq x_1 < x_2 \leq 1$***)***

In this scenario, the profit of the online store is

$$\prod_1^1 = [1 - \frac{p_1 - \beta p_2}{t(1-\beta\lambda)} + \frac{\beta(p_2 - p_1)}{\lambda t}](p_1 - w)\,. \tag{A. 5}$$

The profit function is concave in p_1. The interior best response of the online store can be got by setting $\partial\prod_1^1/\partial p_1 = 0$ i.e.

$$\Omega_1^1 = \frac{\beta\gamma_1 p_2 + \gamma_2 w + (1-\beta)\lambda t}{2\gamma_2}\,. \tag{A. 6}$$

In this scenario, the profit of the physical store is

$$\prod_2^1 = \beta[\frac{p_1 - \beta p_2}{t(1-\beta\lambda)} - \frac{p_2 - p_1}{t\lambda}](p_2 - w)\,. \tag{A. 7}$$

The profit function is concave in p_2. The interior best response of the physical store can be got by setting $\partial\prod_2^1/\partial p_2 = 0$ i.e.

$$\Omega_2^1 = \frac{\gamma_1 p_1 + w}{2}\,. \tag{A. 8}$$

Scenario P-O ($x_1 \leq 0$ ***and*** $0 < x_2 < 1$***)***

In this scenario, the profit of the online store is

$$\prod_1^2 = [1 - \frac{p_1 - \beta p_2}{t(1-\beta\lambda)}](p_1 - w). \tag{A. 9}$$

The profit function is concave in p_1. The interior best response of the online store can be got by solving $\partial \prod_1^2 / \partial p_1 = 0$ i.e.

$$\Omega_1^2 = \frac{\beta p_2 + w + t(1-\beta\lambda)}{2}. \tag{A. 10}$$

In this scenario, the profit of the physical store is:

$$\prod_2^2 = \frac{\beta(p_1 - \beta p_2)}{t(1-\beta\lambda)}(p_2 - w). \tag{A. 11}$$

The profit function is concave in p_2. The interior best response of the physical store can be got by setting $\partial \prod_2^2 / \partial p_2 = 0$ i.e.

$$\Omega_2^2 = \frac{p_1 + \beta w}{2\beta}. \tag{A. 12}$$

- **Best responses**

At prices that allow both stores to make positive profits (i.e., when price or demand for neither firm is zero), a store's best response in each scenario might lie either in the interior of the scenario (shown in Table A.2) or on the boundary separating that scenario from its adjacent scenario (Boundary S-P or Boundary P-O). Moreover, for certain prices, the store might have a choice of responding in one of two scenarios, in which case it will select the scenario that yields higher profits. Consequently, a store's response might jump abruptly from (the interior of) one scenario to the other as the competitor's price increases.

The online store's best responses

Local best responses

In Scenario S-P, the online store's interior best response is $p_1 = \Omega_1^0$. Substituting this value of p_1 to the Boundary S-O requirement $p_2 < \gamma_1 p_1$, we get $p_2 < P_2^{0,S-O}$, where $P_2^{0,S-O} = \frac{\gamma_1 w}{2-\gamma_1}$. Substituting this value of p_1 to the Boundary S-P requirement $p_2 < \frac{p_1 - t(\gamma_1 - \lambda)}{\beta}$, we get $p_2 < P_2^{0,S-P}$, where $P_2^{0,S-P} = \frac{w - 2(\gamma_1 - \lambda)t}{2\beta - 1}$. Substituting this value of p_1 to the Boundary P-O requirement $p_2 > p_1$, we get $p_2 > w$, which is self-satisfied when the physical store maximizes its profit. We find that $p_1 = \Omega_1^0$ intersects Boundary P-O at $p_2 = w$, and either intersects Boundary S-O at $p_2 = \frac{\gamma_1 w}{1-\lambda+\beta\lambda}$ or intersects Boundary S-P at $p_2 = \frac{w - 2(\gamma_1 - \lambda)t}{2\beta - 1}$. In this scenario, the online store's local best response is in the interior for $p_2 < \min\{P_2^{0,S-O}, P_2^{0,S-P}\}$, lies on Boundary S-P if $\min\{P_2^{0,S-O}, P_2^{0,S-P}\} \le p_2 \le \frac{\gamma_1 t}{1-\beta}$, and lies on Boundary S-O if $p_2 > \min\left\{\frac{\gamma_1 t}{1-\beta}, P_2^{0,S-O}\right\}$.

In Scenario S-P-O, the online store's interior best response is $p_1 = \Omega_1^1$. Substituting this value of p_1 in the Boundary S-O requirement $p_2 < \gamma_1 p_1$, we get $p_2 < P_2^{1,S-O}$, where $P_2^{1,S-O} = \frac{\gamma_1[\gamma_2 w + (\gamma_1 - \lambda)\lambda t]}{2\gamma_2 - \beta\gamma_1^2}$. Substituting this value of p_1 in the Boundary P-O requirement $p_2 > p_1$, we get $p_2 > P_2^{1,VP-NO}$, where $P_2^{1,P-O} = \frac{\gamma_2 w + (\gamma_1 - \lambda)\lambda t}{\gamma_2 + \gamma_1 - 1}$. Substituting this value of p_1 in the Boundary S-P requirement $p_2 > \frac{p_1 - t(\gamma_1 - \lambda)}{\beta}$, we get $p_2 > P_2^{1,S-P}$, where $P_2^{1,S-P} = \frac{\gamma_2 w - (\gamma_1 - \lambda)(2\gamma_2 - \lambda)t}{(2\gamma_2 - \gamma_1)\beta}$. We find that $p_1 = \Omega_1^1$ intersects Boundary S-O at $p_2 = P_2^{1,S-O}$ and that it either intersects Boundary P-O at $p_2 = P_2^{1,P-O}$ or intersects Boundary S-P at $p_2 = P_2^{1,S-P}$. In this scenario, the online store's local best response is in the interior for $\max\{P_2^{1,P-O}, P_2^{1,S-P}\} < p_2 < P_2^{1,S-O}$, lies on Boundary S-O for $p_2 \ge P_2^{1,S-O}$, lies on Boundary S-P for $\frac{t(\gamma_1 - \lambda)}{1-\beta} < p_2 \le max\{P_2^{1,P-O}, P_2^{1,S-P}\}$, and lies on Boundary P-O for $p_2 \le max\{P_2^{1,P-O}, \frac{t(\gamma_1 - \lambda)}{1-\beta}\}$.

In Scenario P-O, the online store's interior best response is $p_1 = \Omega_1^2$. Substituting this value of p_1 in the Boundary P-O requirement $p_2 < p_1$, we get $p_2 < P_2^{2,P-O}$, where $P_2^{2,P-O} = \frac{w+(\gamma_1-\lambda)t}{2-\beta}$. Substituting this value of p_1 in the Boundary S-P requirement $p_2 > \frac{p_1 - t(\gamma_1-\lambda)}{\beta}$, we get $p_2 > P_2^{2,S-P}$, where $P_2^{2,S-P} = \frac{w - t(\gamma_1-\lambda)}{\beta}$. We find that $p_1 = \Omega_1^2$ intersects Boundary P-O at $p_2 = P_2^{2,P-O}$ and intersects Boundary S-P at $p_2 = P_2^{2,S-P}$. In this scenario, the online store's local best response is in the interior for $P_2^{2,S-P} < p_2 < P_2^{2,P-O}$, lies on Boundary P-O for $p_2 \ge P_2^{2,P-O}$, and lies on Boundary S-P for $p_2 \le P_2^{2,S-P}$.

Discontinuity in the online store's response between Scenario S-P-O and Scenario S-P, and between Scenario S-P-O and Scenario P-O

The online store chooses between Scenario S-P-O and Scenario P-O for $p_2 \le \frac{t(\gamma_1-\lambda)}{1-\beta}$ and chooses between Scenario S-P-O and Scenario S-P for $\frac{t(\gamma_1-\lambda)}{1-\beta} < p_2 < \frac{\gamma_1 t}{1-\beta}$.

We first demonstrate the discontinuity in the online store's response between Scenario S-P-O and Scenario S-P and identify a threshold price above (below) which the Scenario S-P (S-P-O) interior response is more profitable. Recall that the inner best response of Scenario S-P-O intersects Boundary S-P at $p_2 = P_2^{1,S-P}$ and that Scenario S-P intersects Boundary S-P at $p_2 = P_2^{0,S-P}$. If $w \ge \frac{t(\gamma_1-\lambda)}{1-\beta}$ (i.e., $P_2^{1,S-P} \le P_2^{0,S-P}$), the online store responds on Boundary P-O for $P_2^{1,S-P} \le p_2 \le P_2^{0,S-P}$. If $w < \frac{t(\gamma_1-\lambda)}{1-\beta}$ (i.e., $P_2^{1,S-P} > P_2^{0,S-P}$), the online store's Scenario S-P-O and Scenario S-P responses are never simultaneously on Boundary S-P. In addition, for all prices at which one of these responses is on the boundary, the other scenario's interior response is more profitable.

Lemma A.1 *When the online store can respond in the interior of either Scenario S-P-O or Scenario S-P (i.e.,* $P_2^{0,S-P} < p_2 < P_2^{1,S-P}$*) and* $w < \frac{t(\gamma_1-\lambda)}{1-\beta}$*, the online store's Scenario S-P interior response is more (less) profitable than the Scenario S-P-O interior response if and only if* $p_2 > \hat{p}_2^{01}$ *(* $p_2 < \hat{p}_2^{01}$*), where*

$$\hat{p}_2^{01} = \frac{\beta\gamma_2 w - \beta(\gamma_1-\lambda)\gamma_1 t - |(\gamma_1-\lambda)t - (1-\beta)w| \sqrt{(\gamma_2-\lambda)\gamma_2}}{(2\beta\gamma_1 - 1)\beta}$$

Proof of Lemma A.1

Let $\Delta\Pi_1^{01} = \Pi_1^0 - \Pi_1^1$. We have $\Delta\Pi_1^{01}(p_2 = P_2^{0,S-P}) < 0$, $\Delta\Pi_1^{01}(p_2 = P_2^{1,S-P}) > 0$, and $\frac{\partial \Delta\Pi_1^{01}}{\partial p_2} > 0$. There exists a threshold $\hat{p}_2^{01}$ such that $\Delta\Pi_1^{01} > 0$ if and only if $p_2 > \hat{p}_2^{01}$ and $\Delta\Pi_1^{01} < 0$ if and only if $p_2 < \hat{p}_2^{01}$, where $\hat{p}_2^{01}$ is determined by the smaller root of the quadratic equation $\Delta\Pi_1^{01} = 0$. □

We then demonstrate the discontinuity in the online store's response between Scenario S-P-O and Scenario P-O and identify a threshold price above (below) which the Scenario S-P-O (P-O) interior response is more profitable. Recall that the inner best response of Scenario S-P-O intersects Boundary P-O at $p_2 = P_2^{1,P-O}$ and that of Scenario P-O intersects Boundary P-O at $p_2 = P_2^{2,P-O}$. Because $P_2^{2,P-O} > P_2^{1,P-O}$ for $w < p_2 \le \frac{t(\gamma_1-\lambda)}{1-\beta}$, the online store's Scenario S-P-O and Scenario P-O responses are never simultaneously on Boundary P-O. And for all prices at which one of these responses is on the boundary, the other scenario's interior response is more profitable. □

Lemma A.2 *When the online store can respond in the interior of either Scenario S-P-O or Scenario P-O (i.e.,* $P_2^{1,P-O} < p_2 < P_2^{2,P-O}$*), the online store's Scenario S-P-O interior response is more (less) profitable than the Scenario P-O interior response if and only if* $p_2 > \hat{p}_2^{12}$ *(* $p_2 < \hat{p}_2^{12}$ *), where*

$$\hat{p}_2^{12} = \frac{\gamma_2 w - (\gamma_1-\gamma_2)\lambda t + |(\gamma_1-\lambda)t - (1-\beta)w| \sqrt{\gamma_2\lambda}}{(2\gamma_1 - 1)\beta}$$

Proof of Lemma A.2:

Let $\Delta\Pi_1^{12} = \Pi_1^1 - \Pi_1^2$. We have $\Delta\Pi_1^{12}(p_2 = P_2^{1,P-O}) < 0$, $\Delta\Pi_1^{12}(p_2 = P_2^{2,P-O}) > 0$, and $\frac{\partial \Delta\Pi_1^{12}}{\partial p_2} > 0$. There exists a threshold $\hat{p}_2^{12}$ such that $\Delta\Pi_1^{12} > 0$ if and only if $p_2 > \hat{p}_2^{12}$ and $\Delta\Pi_1^{12} < 0$ if and only if $p_2 < \hat{p}_2^{12}$, where $\hat{p}_2^{12}$ is determined by the bigger root of the quadratic equation $\Delta\Pi_1^{12} = 0$. □

Overall best response

The preceding two lemmas, together with our previous observations about the online store's best responses, establish the validity of the online store's overall best response function (denoted by Ω_1), specified below.

If $\frac{t}{w} > \frac{1-\beta}{1-\beta\lambda}$*, the online store's best response price* Ω_1 *is*

(i) in the interior of Scenario P-O i.e. $\Omega_1 = \frac{\beta p_2 + w + (1-\beta\lambda)t}{2}$ *for any* $P_2^{2,S-P} < p_2 \le \hat{p}_2^{12}$*;*

(ii) in the interior of Scenario S-P-O i.e. $\Omega_1 = \frac{\beta\gamma_1 p_2 + \gamma_2 w + (1-\beta)\lambda t}{2\gamma_2}$ *for any* $\hat{p}_2^{12} < p_2 < \hat{p}_2^{01}$*;*

(iii) in the interior of Scenario S-P i.e. $\Omega_1 = \frac{p_2 + w}{2}$ *for any* $\hat{p}_2^{01} \le p_2 < \min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$*;*

(iv) On Boundary S-P i.e. $\Omega_1 = \beta p_2 + t(1-\beta\lambda)$ *for any* $\min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\} \le p_2 \le \frac{\gamma_1 t}{1-\beta}$*; and*

(v) on Boundary S-O i.e. $\Omega_1 = \frac{p_2}{\gamma_1}$ *for any* $p_2 > \min\left\{\frac{\gamma_1 t}{1-\beta}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$ *or* $p_2 \le P_2^{2,S-P}$.

If $\frac{t}{w} \le \frac{1-\beta}{1-\beta\lambda}$*, the online store's best response price* Ω_1 *is*

(i) in the interior of Scenario P-O i.e. $\Omega_1 = \frac{\beta p_2 + w + (1-\beta\lambda)t}{2}$ *for any* $p_2 \le \hat{p}_2^{12}$*;*

(ii) in the interior of Scenario S-P-O i.e. $\Omega_1 = \frac{\beta\gamma_1 p_2 + \gamma_2 w + (1-\beta)\lambda t}{2\gamma_2}$ *for any* $\hat{p}_2^{12} < p_2 < P_2^{1,S-O}$*;*

(iii) in the interior of Scenario S-P i.e. $\Omega_1 = \frac{p_2 + w}{2}$ *for any* $P_2^{0,S-P} < p_2 < \min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$*;*

(iv) on Boundary S-P i.e. $\Omega_1 = \beta p_2 + t(1-\beta\lambda)$ *for any* $P_2^{1,S-O} \le p_2 \le P_2^{0,S-P}$ *and* $\min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\} \le p_2 \le \frac{\gamma_1 t}{1-\beta}$*; and*

(v) on Boundary S-O i.e. $\Omega_1 = \frac{p_2}{\gamma_1}$ *for any* $p_2 > \min\left\{\frac{\gamma_1 t}{1-\beta}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$ *or* $p_2 \le P_2^{2,S-P}$.

The physical store's best responses

Local best responses

In Scenario S-P, the physical store's interior best response is $p_2 = \Omega_2^0$. Substituting this value of p_2 in the Boundary S-O requirement $p_2 < \gamma_1 p_1$, we get $p_1 > P_1^{0,S-O}$, where $P_1^{0,S-O} = \frac{w + \lambda t}{2\gamma_1 - 1}$. Substituting this value of p_2 in the Boundary S-P requirement $p_2 < \frac{p_1 - t(\gamma_1 - \lambda)}{\beta}$, we get $p_1 > P_1^{0,S-P}$, where $P_1^{0,S-P} = \frac{\beta w + (2-\lambda\beta)t}{2-\beta}$. Substituting this value of p_2 in the Boundary P-O requirement $p_2 > p_1$, we get $p_1 < P_1^{0,P-O}$, where $P_1^{0,P-O} = w + \lambda t$. We find that $p_2 = \Omega_2^0$ intersects Boundary P-O at $p_1 = P_1^{0,P-O}$ and either intersects Boundary S-O at $p_1 = P_1^{0,S-O}$ or intersects Boundary S-P at $p_1 = P_1^{0,S-P}$. In this scenario, the physical store's local best response is in the interior for $\max\left\{P_1^{0,S-O}, P_1^{0,S-P}\right\} < p_1 < P_1^{0,P-O}$, lies on Boundary S-O if $\frac{t}{1-\beta} < p_2 \le \max\left\{P_1^{0,S-O}, P_1^{0,S-P}\right\}$, lies on Boundary S-P if $p_2 \le \min\left\{\frac{t}{1-\beta}, P_1^{0,S-P}\right\}$, and lies on Boundary P-O if $p_1 \ge P_1^{0,P-O}$.

In Scenario S-P-O, the physical store's interior best response is $p_2 = \Omega_2^1$. Substituting this value of p_2 in the Boundary S-O requirement $p_2 < \gamma_1 p_1$, we get $p_1 > P_1^{1,S-O}$, where $P_1^{1,S-O} = \frac{w}{\gamma_1}$. Substituting this value of p_2 in the Boundary P-O requirement $p_2 > p_1$, we get $p_1 < P_1^{1,P-O}$, where $P_1^{1,P-O} = \frac{w}{2-\gamma_1}$. Substituting this value of p_2 in the Boundary S-P requirement $p_2 > \frac{p_1 - t(\gamma_1 - \lambda)}{\beta}$, we get $p_1 < P_1^{1,S-P}$, where $P_1^{1,S-P} = \frac{\beta w + 2(\gamma_1 - \lambda)t}{2 - \beta\gamma_1}$. We find that $p_2 = \Omega_2^1$ intersects Boundary S-O at $p_1 = P_1^{1,S-O}$ and either intersects Boundary P-O at $p_1 = P_1^{1,P-O}$ or intersects Boundary S-P at $p_1 = P_1^{1,S-P}$. In this scenario, the physical store's local best response is in the interior for $P_1^{1,S-O} < p_2 < \min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\}$, lies on Boundary S-O for $p_1 \le P_1^{1,S-O}$, lies on Boundary P-O for $\min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\} \le p_2 < \frac{t(\gamma_1 - \lambda)}{1-\beta}$, and lies on Boundary S-P for $p_2 \ge \max\left\{P_1^{1,S-P}, \frac{t(\gamma_1 - \lambda)}{1-\beta}\right\}$.

In Scenario P-O, the physical store's interior best response is $p_2 = \Omega_2^2$. Substituting this value of p_2 in the Boundary P-O requirement $p_2 < p_1$, we get $p_1 > P_1^{2,P-O}$, where $P_1^{2,P-O} = \frac{\beta w}{2\beta - 1}$. Substituting this value of p_1 in the Boundary S-P requirement $p_2 > \frac{p_1 - t(1-\beta\lambda)}{\beta}$, we get $p_1 < \mathrm{P}_1^{2,\mathrm{S\text{-}O}}$, where $P_1^{2,S-O} = \beta w + 2t(2-\gamma_1)$. We find that $p_1 = \Omega_1^2$ intersects Boundary P-O at $p_1 = P_1^{2,P-O}$ and intersects Boundary S-P at $p_1 = P_1^{2,S-O}$. In this scenario, the physical store's local best response is in the interior for $P_1^{2,P-O} < p_1 < \mathrm{P}_1^{2,\mathrm{S\text{-}O}}$, lies on Boundary P-O for $p_2 \le P_1^{2,P-O}$, and lies on Boundary S-P for $p_1 \ge P_1^{2,S-O}$.

Overall best response

Given p_1, if $p_1 \le P_1^{2,P-O}$, the physical store reacts in the interior of Scenario P-O or Scenario S-P-O; if $P_1^{2,P-O} < p_1 \le \frac{t}{1-\beta}$, the physical store reacts in the interior of Scenario S-P-O or Scenario S-P; if $p_1 > \frac{t}{1-\beta}$, the physical store reacts in the interior of Scenario S-P. The preceding three observations imply that when $\min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\} \le p_2 \le P_1^{2,P-O}$, the retailer's responses in Scenarios P-O and S-P-O are both on Boundary P-O; and when $P_1^{2,S-O} \le p_2 \le \min\left\{\frac{t}{1-\beta}, P_1^{0,S-P}\right\}$, the retailer's responses in Scenarios P-O and S-P are both on Boundary S-P. Moreover, the interior response yields higher profit for the retailer.

Together with our previous observations about the online store's best responses, the online store's best response price Ω_1 is:

- *(i)* *on Boundary S-O i.e.,* $\Omega_2 = \gamma_1 p_1$ *for any* $p_1 \le P_1^{1,S-O}$ *or* $\frac{t}{1-\beta} < p_2 \le \max\left\{P_1^{0,S-O}, P_1^{0,S-P}\right\}$*;*
- *(ii)* *in the interior of Scenario S-P-O i.e.,* $\Omega_2 = \frac{\gamma_1 p_1 + w}{2}$ *for any* $P_1^{1,S-O} < p_2 < \min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\}$*;*
- *(iii)* *on Boundary P-O i.e.,* $\Omega_2 = p_1$ *for any* $\min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\} \le p_2 \le P_1^{2,P-O}$ *or* $p \ge P_1^{0,P-O}$*;*
- *(iv)* *in the interior of Scenario P-O i.e.,* $\Omega_2 = \frac{p_1 + \beta w}{2\beta}$ *for any* $P_1^{2,P-O} < p_1 < P_1^{2,S-O}$*;*
- *(v)* *on Boundary S-P i.e.,* $\Omega_2 = \frac{p_1 - t(1-\beta\lambda)}{\beta}$ *for any* $P_1^{2,S-O} \le p_1 \le \min\left\{\frac{t}{1-\beta}, P_1^{0,S-P}\right\}$*; and*
- *(vi)* *in the interior of Scenario S-P i.e.* $\Omega_2 = \frac{p_1 + w + \lambda t}{2}$ *for any* $\max\left\{P_1^{0,S-O}, P_1^{0,S-P}\right\} < p_1 < P_1^{0,P-O}$.

- **Equilibrium**

Inner equilibrium

We determine the necessary and sufficient conditions, on the parameter range, for the interior equilibrium to hold.

Scenario S-P

Scenario S-P inner equilibrium can be obtained by setting $p_1 = \Omega_1^0$ and $p_2 = \Omega_2^0$ simultaneously, which should satisfy the following two conditions: (i) $\frac{t}{w} > \frac{1-\beta}{1-\beta\lambda}$ and $\hat{p}_2^{01} \le p_2 < \min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$, or $\frac{t}{w} \le \frac{1-\beta}{1-\beta\lambda}$ and $P_2^{0,S-P} < p_2 < \min\left\{P_2^{0,S-P}, \frac{\gamma_1 w}{2-\gamma_1}\right\}$; and (ii) $\max\left\{P_1^{0,S-O}, P_1^{0,S-P}\right\} < p_1 < P_1^{0,P-O}$.

The necessary and sufficient condition for Scenario S-P inner equilibrium occurs is $\frac{t}{w} < U_0 = \frac{3(1-\beta)}{3-\beta\lambda-\lambda}$, where the upper bound is determined by the Boundary S-P requirement.

Scenario S-P-O

The Scenario S-P-O inner equilibrium can be obtained by setting $p_1 = \Omega_1^1$ and $p_2 = \Omega_2^1$ simultaneously, which should satisfy the following two conditions: (i) $\frac{t}{w} > \frac{1-\beta}{1-\beta\lambda}$ and $\hat{p}_2^{12} < p_2 < \hat{p}_2^{01}$, or $\frac{t}{w} \le \frac{1-\beta}{1-\beta\lambda}$ and $\hat{p}_2^{12} < p_2 < P_2^{1,S-O}$; and (ii) $P_1^{1,S-O} < p_2 < \min\left\{P_1^{1,P-O}, P_1^{1,S-P}\right\}$.

The necessary and sufficient condition of Scenario S-P-O inner equilibrium is $L_1 < \frac{t}{w} < U_1$, where $L_1 = \frac{\overline{\beta}[(3-\lambda\beta)\gamma_2 - \lambda]}{(\gamma_1-\lambda)[(3+\beta\lambda)\beta\gamma_1 - 2(2\beta-1)\lambda]}$ and $U_1 = \frac{\overline{\beta}[(\gamma_1-\lambda)\lambda(4-2\beta+3\beta\gamma_1)\gamma_2 - (4\gamma_2-\beta\gamma_1^2)\sqrt{\lambda\gamma_2}]}{(\gamma_1-\lambda)[(4-4\beta+\beta\gamma_1)\lambda\gamma_2 - (4\gamma_2-\beta\gamma_1^2)\sqrt{\lambda\gamma_2}]}$. Note that the upper bound is determined by the requirement that the online store does not jump to Scenario P-O $p_2 > \hat{p}_2^{12}$ and the lower bound is determined by the Boundary S-P requirement.

Scenario P-O

The Scenario P-O inner equilibrium can be obtained by setting $p_1 = \Omega_1^2$ and $p_2 = \Omega_2^2$ simultaneously, which should satisfy the following two conditions: (i) $p_2 \le \hat{p}_2^{12}$ and (ii) $P_1^{2,P-O} < p_1 < P_1^{2,S-O}$.

The necessary and sufficient condition of Scenario P-O inner equilibrium is $\frac{t}{w} > L_2 = \frac{(1-\beta)(\gamma_1 + 3\sqrt{\lambda\gamma_2})}{(1-\lambda\beta)(4-5\gamma_1+3\sqrt{\lambda\gamma_2})}$ if $L_2 > 0$ i.e. $\beta > \frac{1+8\lambda}{17\lambda}$. The equilibrium never occurs in the interior of Scenario P-O if $\beta \le \frac{1+8\lambda}{17\lambda}$.

Boundary equilibrium

If $\min\{U_0, L_1\} < \frac{t}{w} < L_1$, according to the best responses of each store, the physical store sets the price on Boundary S-P, and the online store reacts in the interior of Scenario S-P. The equilibrium is obtained by solving equations $p_1 = \Omega_1^0$ and $p_2 = \frac{p_1 - t(1-\beta\lambda)}{\beta}$ simultaneously.

If $U_1 \le \frac{t}{w} \le L_2$, according to the best responses, the physical store sets the price on Boundary P-O, and the online store reacts in the interior of Scenario P-O. The equilibrium is obtained by solving equations $p_1 = \Omega_1^2$ and $p_2 = p_1$ simultaneously.

Overall equilibrium

The overall equilibrium is given in Table 1.

Proof of Proposition 3-5

Proposition 3-5 can be obtained by differentiating the equilibrium prices and profits in each scenario with respect to t, β and λ respectively. The details are omitted.