ADVERTISING OR BROKERAGE MODEL FOR SOCIAL PLATFORMS WITH A COMMERCE FEATURE

Guofang Nan Management School Hainan University No. 58 Renmin Avenue, Haikou, P.R. China 570228 <u>gfnan@tju.edu.cn</u>

Chenyu Chu College of Management and Economics Tianjin University No.92 Weijin Road, Nankai District, Tianjin, P.R. China 300072 chucy1012@tju.edu.cn

Zhiyong Li^{*} ¹State Key Laboratory of Media Convergence and Communication ²School of Economics and Management Communication University of China No.1 Dingfuzhuang East Street, Chaoyang District, Beijing, P.R.China 100024 <u>zyli@cuc.edu.cn</u>

Minqiang Li College of Management and Economics Tianjin University No.92 Weijin Road, Nankai District, Tianjin, P.R. China 300072 <u>mqli@tju.edu.cn</u>

ABSTRACT

This study analyzes the revenue-model decision of social platforms with a commerce feature. The social platforms can adopt either an advertising model or a brokerage model. We first address the social platform's optimal pricing strategies under a given revenue model and then present the condition under which each revenue model is optimal. We find that, when the fixed fee is very low (high), the advertising (brokerage) model is always optimal for the social platform. When the fixed fee is medium, the advertising model is optimal. Interestingly, we find that the disutility of advertisements is very low; otherwise, the brokerage model is optimal. Interestingly, we find that the condition under which each revenue model is optimal also enables the social platform to be allocated more profits from the supply chain. This study enriches the theoretical foundation on the revenue model decision of the social platform which connects three (when the brokerage model is adopted) or four (when the advertising model is adopted) different groups of agents. Our findings provide insights into the revenue-model decision of a social platform with commerce.

Keywords: Social platform; Commerce feature; Advertising; Brokerage; Two-sided market

1. Introduction

Emerging Web 2.0 technologies and social media have changed how business is conducted as well as the way that people communicate, collaborate, and live [Busalim & Hussin 2016]. Given the rapid development of social media, social platforms are constantly enriching and improving features to meet the growing demands of users, hoping to attract more users and to increase user stickiness. Social platforms also try to exploit the installed base of users to obtain more revenue by expanding their profit channels. The development of e-commerce also has made

^{*} Corresponding Author

online shopping increasingly popularly. The combination of social networking and e-commerce has become a pursuit of platforms, which also promotes the formation of social commerce.

Many social platforms incorporate a commerce feature for higher profits. For example, Facebook launched Marketplace to facilitate the buying and selling of products between consumers and sellers [Lee 2018]. As of October 2016. Marketplace has grown at a rate of 18 million new listings per month, and search volume had increased threefold. A report found that more than 800 million people used Marketplace each month to buy or sell across the 71 countries where it was operational, including one out of three U.S. Facebook users [Perez 2018]. The Little Red Book is a platform to share and read product reviews and lifestyle tips across a host of product categories. The unique aspect of this platform is community and user experience. It achieved 85 million monthly active users in the first half of 2019, of which a high proportion were women from China's first- and second-tier cities [Norris 2019]. There is also a specific shopping area in the Little Red Book platform. For Pinterest, it launched a smallbusiness shop on its site as it leaned into shopping in 2019. "Pinterest Shop" enables users to shop from 17 small businesses. The Pinterest platform is particularly popular with mothers, who make up 80% of its total U.S. audience, according to a Comscore study [Feiner 2019]. These social platforms, however, choose different models to profit from the commerce feature. To generate more profits, Facebook began to allow a select number of advertisers to run ads in Marketplace in 2017. Then, without making a formal announcement, Facebook opened the ability to run ads in Marketplace to all U.S. advertisers in January 2018 and expanded this feature to Canada in May 2018 [Gesenhues 2018]. Instead of profiting from advertisers, Little Red Book and Pinterest generate revenue from sellers in the commerce module, as merchants need to pay a deposit fee and commission fee for selling products.

An important observation is that a social platform with a commerce feature can adopt either an advertising model or a brokerage model. In the advertising model, consistent with the revenue model deployed by Facebook, sellers can sell their products in Marketplace for free, and Facebook generates profit by attracting advertisers to advertise their products in Marketplace. The brokerage model is consistent with the revenue model adopted by Little Red Book and Pinterest, which also launched a specific shopping area, in which profit is generated by charging a transaction fee and a fixed fee to sellers. The main difference between the advertising and brokerage models is as follows. If a social platform adopts an advertising model, the addition of consumers has a positive effect on the advertiser, whereas an increase in the number of advertisers reduces consumer's utility. Consequently, the platform should be cautious about the tradeoff between the demands of different groups when it sets an advertising fee to balance the consumer demand and the seller's and the platform's profits. Thus, it is not clear which revenue model is optimal for a platform to adopt and how a social platform cam profit from the commerce feature. This study develops an analytical framework to establish guidelines for the social platform. We focus on determining the better revenue model for the social platform and analyzing the effect of the revenue rate of advertisers on platforms' decision making.

Regarding the adoption of an ad revenue model for the advertising revenue of the platform, two types models are relevant: the performance-based model and the cost-per-thousand-impressions (CPM) (i.e., cost-per-mile) model. Performance-based revenue models take into account user ad aversion and charge advertisers on the basis of user actions. For example, in cost-per-click (CPC)—a highly successful and popular performance-based model— advertisers pay only when their ads generate clicks [Lin et al. 2012]. Our research considers the setting in which the platform can choose a CPM or CPC ad revenue model for its ad service when the advertising model is adopted. We first compare the two kinds of advertising models with the brokerage model and then derive the optimal revenue model for the social platform.

The study considers a monopoly market for a social platform with the commerce feature. The platform adopts either an advertising model or a brokerage model. We determine the better revenue model for the social platform and analyze the impacts of several factors (e.g., the revenue rate of advertisers) on the choice of strategy. Specifically, we answer the following questions: Which revenue model, i.e., the advertising model or the brokerage model, should be chosen when a social platform decides to provide the commerce feature? How do the characteristic of advertisers affect the choice of the revenue model? How does the revenue rate of advertisers affect the decision to charge an advertising fee under advertising model? What are the effects of the revenue-model decision on customer surplus and profit allocation in the supply chain?

To answer these questions, we develop a game-theoretic model in which a monopolistic social platform intends to provide a commerce feature by which a seller can sell his or her product or service, and consumers can purchase the product or service. As noted, the social platform can generate revenue from an advertising model or a brokerage model. The social platform chooses a CPM or CPC ad revenue model for its ad service under the advertising model. The main contributions of this study concern practical and theoretical aspects. In regard to the provision of a commerce feature (the practical aspect), this study can help the social platforms to choose a better revenue model to generate more profits. Further, our findings can provide social platforms with strategic support to maximize their profits while meeting the demands of consumers and sellers. In terms of the theoretical aspect, this study enriches the theoretical research on social commerce and revenue model decisions. The existing theoretical literature on social commerce mainly focuses on the decision of, at most, three different groups of agents, whereas we consider the decisions of three (when the brokerage model is adopted) or four (when the advertising model is adopted) different groups of agents. Moreover, the existing theoretical literature on revenue-model decision is mainly based on the traditional e-commerce while ignore the social feature. Our study enriches the theoretical foundation on the social platform's revenue-model decision. Most importantly, our theoretical finding implies that the social platform should pay attention to the fixed fee and the interaction between advertisers and consumers when choosing the revenue model, a decision that contributes to the business management.

The main findings are as follows. First, the fixed fee and disutility of advertisements to consumers affect the revenue-model decision of the social platform. Specifically, the social platform makes more profits under the advertising model when the fixed fee is very low. When the fixed fee is medium and the disutility of advertisements is very low, the social platform obtains more profits under the advertising model. The social platform obtains more profits under the brokerage model when the fixed fee is medium and the disutility of advertisements is very high. The social platform prefers the brokerage model when the fixed fee is revenue from the ad service. Thus, the situation in which the advertising model is always an optimal choice does not appear. We also analyze the influence of the revenue rate of advertisers with high revenue rate increases. As the revenue rate of advertisers with a low revenue rate increases, the platform prefers the advertising model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high but prefers the brokerage model when the value of the commerce feature is very high.

Second, the impact of the revenue rate of advertisers on the decision of the advertising fee under the advertising model. In particular, as the revenue rate of advertisers with a high revenue rate increases, the optimal advertising fee to advertisers with a high revenue rate increases, while the optimal advertising fee to advertisers with a low revenue rate decreases. Similarly, the optimal advertising fee to advertisers with a low revenue rate increases with the revenue rate of advertisers with a low revenue rate. Interestingly, as the revenue rate of advertisers with a high revenue rate increases, the advertisers with a low revenue rate advertisers with a high revenue rate increases, the advertising fee to advertisers with a low revenue rate of advertisers with a high revenue rate increases, the advertising fee to advertisers with a low revenue rate depends on the composition of different types of advertisers and the difference in the revenue rate between advertisers with a high revenue rate.

Third, the customer surplus is higher under the advertising model when the value of the commerce feature is very high. When the value of the commerce feature is very low, the customer surplus is higher under the advertising model if the disutility of advertisements is very low; otherwise, the customer surplus is higher under the brokerage model. Interestingly, when the social platform adopts the CPC advertising model, the customer surplus is higher under the disutility of advertisements is very high.

Finally, the social platform could be allocated more profits from the supply chain under the advertising model when the fixed fee is very low. When the fixed fee is medium and the disutility of advertisements is very low, the social platform could be allocated more profits under the advertising model. The social platform could be allocated more profits under the fixed fee is medium and the disutility of advertisements is very high. The social platform could be allocated more profits under the brokerage model when the fixed fee is medium and the disutility of advertisements is very high. The social platform could be allocated more profits under the brokerage model when the fixed fee is very high. Interestingly, when the social platform deploys the CPC ad revenue model, it makes more profits under the brokerage model; thus, the situation in which the platform could always be allocated more profits under the advertising model does not appear.

The remainder of the study is organized as follows. Section 2 provides a review of the related literature. Section 3 presents the setup of our basic model. Section 4 concerns the optimal revenue model for the social platform and provides an analysis of the influence of exogenous factors on the decisions of platform. Section 5 presents a comparison of customer surplus and profit allocation in the supply chain between the advertising and brokerage models. Section 6 contains an extended model with non-linear disutility of advertisers to consumers. Section 7 provides the conclusion and managerial insights from this study. Section 8 presents directions for future work.

2. Related Literature

Our research is closely related to two streams of the literature: revenue-model decision and social commerce.

2.1 Revenue-model decision

This study contributes to the emerging research literature that focuses on the revenue-model decision. In the process of growth, enterprises always face the difficulty of revenue-model decisions and must choose one that best suits their enterprise development. Thus, many studies on the revenue-model decision provide theoretical support for

business development (e.g., Yang et al. [2017]). For example, Jing et al. [2011] investigate the decisions of the seller and two types of consumers; compare the revenue model of group buying with traditional individual-selling strategies and another popular social interaction scheme, referral rewards programs; and analyze the best revenue model for selling through social interactions. Chen et al. [2016] take eBay and Taobao as examples and compare the two alternative revenue models for generating profit on online consumer-to-consumer platforms; the advertising model and the commission model. Under the advertising model, Chen et al. [2016] consider how sellers promote their products in advertising space, whereas our research considers how extrinsic advertisers show their content on the social platform and analyze the influence of these advertisers' revenue rate on the revenue-model decision of the platform. Li et al. [2020] consider the decisions of social platforms, advertisers, and users; investigate which revenue model (advertising or freemium) the social platform should choose by considering the duopoly competition; and analyze the customer surplus under different revenue models. We further consider the transaction of products on the social platform as well as the situation in which the platform charges different advertising fees to different advertisers. Schiff [2003] investigates the decisions of the platform and two types of consumers, develops a framework that encompasses matching service and platform revenue models that adopt subscription or pertransaction pricing, and analyzes customer surplus under different revenue models. Ryan et al. [2012] consider a setting in which a firm operates an online marketplace through which retailers can sell products directly to consumers and investigate the decisions of the platform, retailer, and consumers. The retailer decides whether to sell the products in the online marketplace of the platform, while the platform decides whether to sell the competitive products to compete with the retailers. Chen et al. [2015] analyze the situation in which large online retailers (such as Amazon and JD.com) decide whether to open their platforms to allow competitors to sell products and investigate the decision of large online retailers, competitors, and consumers. Li et al. [2018] analyze the optimal distribution strategy of enterprise software by taking into account the distinct features of enterprise software for both short- and long-term problems. Dou et al. [2020] investigate a platform's pricing strategy when consumers are categorized into two distinct groups and compare the equilibrium results derived under scenarios with and without consumer categorization.

Our study differs from the aforementioned studies in four key aspects. First, we focus on the revenue-model decision of the social platform that intends to realize a commerce feature. Second, whereas existing literature analyzes the decision of, at most, three groups of agents, our research considers the decisions of four groups of agents: social platform, seller, advertisers, and consumers. Third, we analyze the influence of the characteristics of advertisers on the revenue-model decision of the social platform. Finally, whereas most of the existing literature analyzes customer surplus to enrich the platform's results, we further investigate the supply chain profit allocation under different revenue models.

2.2 Social commerce

Our study also contributes to the literature on social commerce. Given the rapid development of e-commerce and social media, their combination has become a general trend, seen in the emergence of the concept of social commerce. Social commerce refers to the use of social media (or social network) to facilitate user participation in online businesses. [Zhou et al. 2013; Qiu et al. 2020]. Although the concept of social commerce has been increasingly used and has gained interest among scholars beginning in 2010, research on social commerce is still in the early stages of development [Zhang & Wang 2012]. Much of this research, by adopting an empirical method, explores the factors that affect consumers' consumption attitudes on the social platform. For example, by conducting two between-subject experiments, Lopez et al. [2019] investigates how Facebook's "buy buttons" can affect social platform users' shopping-related attitudinal and behavioral responses and how providing users with a safe shopping environment can affect their shopping-related responses. Yahia et al. [2018] analyze the influence of factors such as trust, social support, and the platform's perceived usage on consumers' social commerce intent and explain the interaction between these factors based on the relevant data of Instagram users. Li [2019] applies a stimulusorganism-response model to investigate the influences of social commerce sites on consumers' virtual experiences and their intentions to purchase products. Jiang et al. [2019] consider information support and social presence theory and construct a theoretical model to examine how information support moderates the relationship between different social presence dimensions and trust in social commerce. Rezaeian et al. [2016] investigate the moderating effect of culture and the mediating role of trust in a social network community in regard to social identity, trust transference (familiarity), social influence (intimacy and friendship), cognitive style, and the intention to purchase in the social business environment. Hu et al. [2016] apply the stimulus-organism-response model and use the empirical model to reveal the influence of peer-member characteristics and technical features of a social shopping website on consumers' purchase intentions. Akman et al. [2017] using a survey approach to determine important behavioral factors, such as satisfaction, ethics, trust, enjoyment or ease, social pressure, and awareness, investigate the factors that influence consumer intention toward the adoption of e-commerce. Nan et al. [2017] investigate how much

product uncertainty varied by reviews, and analyze the influence of product uncertainty on consumers' purchasing behavior. Different from the existing research, we establish a game-theory model to investigate the pricing strategy and the revenue-model decision for a platform with social commerce and analyze the influence of the characteristics of advertisers, such as their revenue rate, on the revenue-model decision of the social platform.

Another stream of research on social commerce focuses on the advantages of using social media for the seller. For example, Wang et al. [2019] hold the view that social media is a widely used marketing tool for reaching potential customers who can learn about products and interact with sellers in real-time. A seller's marketing microblogs, however, may backfire by dominating the social space. Thus, Wang et al. [2019] empirically quantify the optimal level of marketing aggressiveness to achieve maximum popularity in social media. Kumar et al. [2013] show that social media can be used to generate growth in sales, return on investment, and positive word of mouth; it can spread brand knowledge further by creating a unique metric to measure the net influence wielded by a user in a social network and a customer influence effect and predict the user's ability to generate the spread of viral information. Ma et al. [2015] find that consumers who frequently communicate in social networks often make similar purchase decisions and that this similarity may stem from a variety of factors. They believe that there is an interaction between consumers on social platforms and prove the importance of social platforms in promoting the product sales of the merchant. Xiang et al. [2016] introduce the parasocial interaction theory to examine the influence of social relationship factors on the formation of impulse-buying behavior and demonstrate that consumers' perceived enjoyment and impulse-buying tendencies affect their impulsive-buying urge significantly. Ye et al. [2013], by conducting empirical research on Mogujie, find that seller reputation seems to have a different impact on the sales price on different platforms. In digital era, virtual brand communities (VBCs) play an important role in building and strengthening companies' brands and in maintaining customer relationships. Wang et al. [2017] collect primary and secondary data from two VBCs to demonstrate the role of regulatory fit in consumers' VBC participation. They find that the regulatory fit between promotion focus and brand ideal self-congruence has significant positive impacts on utilitarian and hedonic benefits.

In conclusion, the existing studies on social commerce mainly focus on consumers' attitudes toward social commerce and explore the factors that influence their attitude. Moreover, they focus on the advantages of using social media for the seller's side. In contrast, this study combines social networking and e-commerce and develops a stylized economic model to analyze the revenue-model decision from the perspective of the social platform that intends to realize a commerce feature.

3. The Basic Model

We consider a monopoly market for a social platform that intends to provide a commerce feature by which sellers can sell their products or services. Similar to Chen et al. [2015], our model considers a setting in which a seller who sells only one kind of product on one side of the social platform and several buyers (i.e., consumers on platform) might purchase the product. In addition, several advertisers can display their contents on a specific space related to the commerce feature of the platform, and consumers can decide whether to consume the product sold on the social platform. The social platform chooses the method of generating profit, using either an advertising model or a brokerage model (denoted as Model B). Moreover, the social platform might choose CPM (denoted as Model M) or CPC (denoted as Model C) ad revenue models for its ad service under the advertising model. Thus, to maximize the profit, the platform must make a series of decisions, including the choice of revenue model, the advertising fee charged to advertisers under the advertising model, and the transaction fee charged to the seller under the brokerage model, to balance the relationship between consumers and sellers or advertisers.

Consumers. There is a continuum of consumers with a unit mass on the social platform. Consumers can decide whether to buy the product on the social platform, and each has at most one unit demand for the product. Note that different consumers may perceive a different feeling of trust and sense of belonging to the social platform [Akman et al. 2017]; thus, consumers are heterogeneous in their preference for shopping on the social platform. We capture this preference heterogeneity via parameter $\theta \sim U[0,1]$. Due to the operational simplicity of the shopping process and the completeness of related functions, each consumer can perceive a value v when shopping on the platform [Yahia et al. 2018]. On the one hand, the more consumers on the social platform, the more valuable and significant information about the product a consumer can get through the social networking service or the online product reviews [Hu et al. 2016]. On the other hand, more consumers could derive more friendship value through the platform will be greater. In addition, several studies consider the network effects to be linear in the size of the user base [Ellison & Fudenberg 2000; Jing 2007; Li & Chen 2012; Etzion & Pang 2014]. Thus, we use γD_i , $i \in \{M, C, B\}$ to denote the social value, where γ denotes the strength of social effect and D_i , $i \in \{M, C, B\}$ denotes the size of consumers on the social platform.

A consumer's utility from a product that displays network effects is usually modeled as a function of the product's intrinsic value and of the number of consumers who use the product [Ellison & Fudenberg 2000; Etzion & Pang 2014]. In this study, we adopt a similar approach and model the value that a consumer obtains from the product as an additive function of the intrinsic value of the product v_0 and the social value γD_i , $i \in \{M, C, B\}$. Therefore, a consumer's utility for consuming products on the social platform is the value that a consumer obtains from the product that adds to the value that a consumer derives from consuming on the social platform θv minus the price of the product $p_i, i \in \{M, C, B\}$. Moreover, following the literature (e.g., Li et al. [2020]), consumers derive a disutility $cQ_i, i \in \{M, C\}$ when advertisers display their content on the social platform, where c represents the displeasure of a consumer from the participation of each advertiser. $Q_i, i \in \{M, C\}$ denotes the number of advertisers on the social platform. In summary, consumer utility can be presented as follows.

 $u_i = v_0 + \theta v + \gamma D_i - cQ_i - p_i, \ i \in \{M, C\},\label{eq:ui}$

 $u_B = v_0 + \theta v + \gamma D_B - p_B.$

When $u_i \ge 0, i \in \{M, C, B\}$, a consumer will consume the product sold on the social platform.

Advertisers. A continuum of advertisers with a unit mass show their content in a space provided on the social platform. Each incurs different fixed costs t to access the platform. The costs satisfy a uniform distribution with support [0,1]. Consistent with existing studies (e.g., Chen et al. [2016], Li et al. [2020]), we do not consider the competition between advertisers, and each advertiser publishes only one advertisement. In practice, different advertisers receive different revenue from an advertisement; thus, we consider two types of advertisers in the model, who pay different advertising fees to the platform. The advertisers with a higher revenue rate β_{H} are H-type advertisers, and the advertisers with a lower revenue rate β_L are L-type advertisers. Different revenue rates may result from different product categories and different costs in the customer-relationship management. The proportion of H-type advertisers is $b \ (0 < b < 1)$, and the proportion of L-type advertisers is 1 - b. We use $\bar{\beta} \ (\bar{\beta} = b\beta_H + b)$ $(1-b)\beta_{l}$ to denote the average revenue rate of two advertisers so that the results are more concise. The difference between the CPC and CPM models is as follows. Under the CPM model, advertisers receive revenue purely on the basis of the number of consumers' adopting the service; thus, advertisers' revenue is $\beta_k D_M, k \in \{H, L\}$. Under the CPC model, however, advertisers generate ad revenue only when consumers click on ads. As shown in Lin et al. [2012], consumers are more likely to click on the ads when the displeasure of a consumer from the participation of each advertiser c is lower. Their actions (e.g., clicks) signal interest, and advertisers' revenue is contingent on such actions. Because consumers do not always click on ads, consumers' probability of clicking on ads should be only a fraction of 1 - c; we denote this probability by $(1 - c)\tau$, where $\tau \in (0,1)$ is the click rate parameter that adjusts a consumer's ad aversion to his or her clicking probability [Lin et al. 2012]. Therefore, the advertisers' revenue under the CPC model is $(1 - c)\tau\beta_k D_c$, $k \in \{H, L\}$. In addition, advertisers pay w_i^k , $i \in \{M, C\}$, $k \in \{H, L\}$ for displaying content. The utility of the social platform's advertisers under the CPM advertising model is given by

 $\begin{array}{l} H\text{-type }(b) \colon U_M^{\overset{*}{H}} = \beta_H D_M - t_M^{\overset{*}{H}} - w_M^H, \\ L\text{-type }(1\text{-}b) \colon U_M^L = \beta_L D_M - t_M^L - w_M^L. \end{array}$

The utility of the social platform's advertisers under the CPC advertising model is given by

H-type (b): $U_c^H = (1 - c)\tau D_c \beta_H - t_c^H - w_c^H$, *L-type* (1-b): $U_c^L = (1 - c)\tau D_c \beta_L - t_c^L - w_c^L$. When $U_i^k \ge 0$, $i \in \{M, C\}, k \in \{H, L\}$, an advertiser will advertise on the social platform.

Platform. Under the advertising model, the seller can sell on the social platform for free, and the platform generates profit when two types of advertisers pay the advertising fee w_i^k , $i \in \{M, C\}$, $k \in \{H, L\}$ for displaying content. Under the brokerage model, the seller pays a transaction fee f for each sale, and a fixed fee s, such as an annual service fee, to the social platform. We can derive the payoff of the social platform Π_i , $i \in \{M, C, B\}$ as follows.

 $\begin{aligned} \Pi_M &= w_M^H Q_M^H + w_M^L Q_M^L, \\ \Pi_C &= w_C^H Q_C^H + w_C^L Q_C^L, \\ \Pi_B &= f D_B + s, \end{aligned}$

where D_B denotes the consumer demand under the brokerage model, and Q_i^k , $i \in \{M, C\}$, $k \in \{H, L\}$ denotes the number of k-type advertisers under model i.

Seller. Under the advertising model, the seller can sell on the social platform for free. Thus, his or her payoff is equal to the total revenue that he or she earns from selling the products. While under the brokerage model, the seller pays the transaction fee f on each sale and a fixed fee s, such as an annual service fee, to the social platform. The payoff a seller derives π_i , $i \in \{M, C, B\}$ is as follows.

$$\pi_M = p_M D_M, \pi_C = p_C D_C,$$

 $\pi_B = p_B D_B - f D_B - s,$

where $D_i, i \in \{M, C, B\}$ denotes the consumer demand under model *i* and $p_i, i \in \{M, C, B\}$ denotes the price of the product under model *i*.

The sequence of decisions proceeds as follows. In the first stage, the platform chooses a revenue model. In the second stage, the platform decides on the advertising fee for different types of advertisers (transaction fee f to seller) under the advertising model (brokerage model), and the seller decides on the price of his or her product. Finally, the demands and profits are realized. The notations are presented in Table 1.

	Table	1:	Descri	ption	of	Notations
--	-------	----	--------	-------	----	-----------

Variable/Symbol	Description
v_0	Intrinsic value of the product
v	Basic value of the social platform's commerce feature
$\theta \sim U[0,1]$	Consumer preferences for shopping on the social platform
$c \ (0 < c < 1)$	Negative utility of each advertiser to the consumers
$\beta_k, k \in \{H, L\}$	Ad revenue rate of k-type advertisers
τ	Click rate parameter
γ	Strength of social effect
$p_i, i \in \{M, C, B\}$	Price of the product under model <i>i</i>
$w_i^k, i \in \{M, C, B\}, k \in \{H, L\}$	Advertising fee to k-type advertisers under model i
f	Transaction fee charged by the platform to the seller for each sale
S	Fixed fee charged to the seller under the brokerage model
<i>t~U</i> [0,1]	Advertiser's fixed cost for accessing the social platform
е	Adjustment parameter of negative utility caused by advertisement
$Q_i, i \in \{M, C\}$	Number of advertisers under model <i>i</i>
$D_i, i \in \{M, C, B\}$	Size of consumers on the social platform under model <i>i</i>
$\pi_i, i \in \{M, C, B\}$	Payoff of the seller under model <i>i</i>
$\Pi_i, i \in \{M, C, B\}$	Payoff of the platform under model <i>i</i>

4. Revenue Model Decision

In this section, we first derive the optimal results under each revenue model and analyze the impact of the revenue rate of advertisers on the advertising fee charged by the social platform under the advertising model. Then, we compare the optimal results formulated under the advertising and brokerage models to recommend the better one for the social platform. Finally, we discuss the influence of the characteristic of advertisers on the revenue-model decision of the social platform.

In the process of obtaining the optimal results, the social platform maximizes its profit by deciding on the optimal advertising fee or transaction fee, while the seller maximizes his or her profit by deciding on the optimal price of his or her product. Based on the best-response function, we can derive the optimal price and the optimal advertising (transaction) fee. Further, by substituting the optimal price and the optimal advertising (transaction) fee into the profit functions, we can obtain the optimal profits of the seller and the social platform. The following lemmas summarize the optimal outcomes.

Lemma 1: When the social platform chooses the CPM advertising model, the optimal advertising fee charged to H-type advertisers is

$$\begin{split} w_{M}^{H^{*}} &= \frac{(v+v_{0})[4(v-\gamma)\beta_{H}+3c(1-b)\beta_{L}\beta_{H}+4cb\beta_{H}^{2}+c(1-b)\beta_{L}^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}; \\ the optimal advertising fee charged to L-type advertisers is \\ w_{M}^{L^{*}} &= \frac{(v+v_{0})[4(v-\gamma)\beta_{L}+3cb\beta_{L}\beta_{H}+4c(1-b)\beta_{L}^{2}+cb\beta_{H}^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}; \\ the optimal price of the products set by the seller is \\ p_{M}^{*} &= \frac{2(v+v_{0})(c\overline{\beta}+v-\gamma)(3c\overline{\beta}+4v-4\gamma)}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}; \\ the optimal profits of the seller and the platform are \\ \pi_{M}^{*} &= \frac{4(v+v_{0})^{2}(c\overline{\beta}+v-\gamma)(3c\overline{\beta}+4v-4\gamma)^{2}}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}}, \\ and \Pi_{M}^{*} &= \frac{(v+v_{0})^{2}(b\beta_{H}^{2}-b\beta_{L}^{2}+\beta_{L}^{2})}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}}; \\ \end{split}$$

and the optimal number of advertisers and consumer demand are

$$Q_{M}^{*} = \frac{(v+v_{0})[2\beta_{L}(2v-2\gamma+c\beta_{L})-b(\beta_{H}-\beta_{L})(c\beta_{H}-5c\beta_{L}-4v+4\gamma)+3b^{2}c(\beta_{H}-\beta_{L})^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}},$$

$$D_{M}^{*} = \frac{(v+v_{0})[2\beta_{L}(2v-2\gamma+c\beta_{L})-b(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]}{2(v+v_{0})(3c\overline{\beta}+4v-4\gamma)},$$

and $D_M^* = \frac{2(v+v_0)(3c\beta+4v-4r)}{8(2v-2v+c\beta_I)(v-v+c\beta_I)-bc(\beta_H-\beta_I)(c\beta_H-17c\beta_I-24v+24v)+9b^2c^2(\beta_H-\beta_I)^2}$.

Proof. All proofs are in the appendix unless indicated otherwise.

Lemma 2: When the social platform chooses the CPC advertising model, the optimal advertising fee charged to H-type advertisers is

 $(1-c)\tau(v+v_0)[4(v-\gamma)\beta_H+3c(1-c)\tau(1-b)\beta_L\beta_H+4c(1-c)\tau b{\beta_H}^2+c(1-c)\tau(1-b){\beta_L}^2]$

 $w_{C}^{H^{*}} = \frac{(1-c)\tau(\nu+\nu_{0})[4(\nu-\gamma)\rho_{H}+3c(1-c)\tau(1-\nu)\rho_{L}\rho_{H}+7c(1-c)\tau\rho_{H}-7c(1-c)\tau\rho_{L}-24\nu+24\nu]}{8[2\nu-2\gamma+c(1-c)\tau\beta_{L}][\nu-\gamma+c(1-c)\tau\beta_{L}]-bc(1-c)\tau(\beta_{H}-\beta_{L})]c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24\nu+24\nu]+9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}};$ the optimal advertising fee charged to L-type advertisers is

 $w_{C}^{L^{*}} = \frac{(1-c)\tau(v+v_{0})[4(v-\gamma)\beta_{L}+3c(1-c)\tau\beta_{L}\beta_{L}\beta_{H}+4c(1-c)\tau(1-b)\beta_{L}^{2}+c(1-c)\tau\beta_{H}^{2}]}{8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}]-bc(1-c)\tau(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24v+24\gamma]+9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}};$ the optimal price of the products set by the seller is

 $p_{c}^{*} = \frac{2(v+v_{0})[c(1-c)\tau\bar{\beta}+v-\gamma][3c(1-c)\tau\bar{\beta}+4v-4\gamma]}{8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}]-bc(1-c)\tau(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24v+24\gamma]+9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}};$ the optimal profits of the seller and the platform are

 $4(v+v_0)^2 [c(1-c)\tau\overline{\beta}+v-\gamma] [3c(1-c)\tau\overline{\beta}+4v-4\gamma]^2$

$$\pi \mathcal{L} = \frac{\{8[2\nu-2\gamma+c(1-c)\tau\beta_L][\nu-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H - \beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24\nu+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H - \beta_L)^2\}^2}{\tau^2(1-c)^2(\nu+\nu_0)^2(b\beta_H^2 - b\beta_L^2 + \beta_L^2)}.$$

and $\Pi_{C}^{*} = \frac{1}{8[2\nu-2\gamma+c(1-c)\tau\beta_{L}][\nu-\gamma+c(1-c)\tau\beta_{L}]-bc(1-c)\tau(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24\nu+24\gamma]+9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}},$ and the optimal number of advertisers and consumer demand are . .

$$Q_{C}^{*} = \frac{(1-c)\tau(v+v_{0})\{2\beta_{L}[2v-2\gamma+c(1-c)\tau\beta_{L}] - b(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-5c(1-c)\tau\beta_{L}-4v+4\gamma] + 3b^{2}c(1-c)\tau(\beta_{H}-\beta_{L})^{2}\}}{8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}},$$

and $D_{C}^{*} = \frac{2(v+v_{0})[3c(1-c)\tau\beta_{H}+4v-4\gamma]}{8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H}-\beta_{L})[c(1-c)\tau\beta_{H}-17c(1-c)\tau\beta_{L}-24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H}-\beta_{L})^{2}},$

Lemma 3: When the social platform chooses the brokerage model, the optimal transaction fee charged by the platform is

 $f^* = \frac{v + v_0}{2};$

the optimal price of the products set by the seller is $p_B^* = \frac{3\hat{v}+3v_0}{4};$

the optimal profits of the seller and the platform are

$$\pi_B^* = \frac{(v+v_0)^2}{16(v-\gamma)} - s,$$

and $\Pi_B^* = \frac{(v+v_0)^2}{8(v-\gamma)} + s;$

and the optimal consumer demand is $v + v_0$ D*

$$D_B^* = \frac{1+r_0}{4(v-\gamma)}.$$

The effect of the revenue rate of advertisers on the advertising fee is summarized by the following proposition. Proposition 1: Under the advertising model, all other parameters being equal,

- (i) the advertising fee to H-type advertisers w_i^H , $i \in \{M, C\}$ increases with the ad revenue rate of H-type advertisers β_H ;
- (ii) there exists thresholds b^* , k^* , and k^{**} ($k^* < k^{**}$) when $\frac{\beta_H}{\beta_I} < k^*$; the advertising fee to L-type advertisers w_i^L , $i \in$ $\{M, C\}$ decreases with the ad revenue rate of H-type advertisers β_H when $\frac{\beta_H}{\beta_L} > k^{**}$; the advertising fee to L-type advertisers w_i^L , $i \in \{M, C\}$ increases with the ad revenue rate of H-type advertisers β_H , when $k^* < \frac{\beta_H}{\beta_L} < k^{**}$; the advertising fee to L-type advertisers w_i^L , $i \in \{M, C\}$ increases with the ad revenue rate of H-type advertisers β_H when $b < b^*$; while the advertising fee to L-type advertisers w_i^L , $i \in \{M, C\}$ decreases with the ad revenue rate of *H*-type advertisers β_H when $b > b^*$;
- (iii) the advertising fee to H-type advertisers w_i^H , $i \in \{M, C\}$ decreases with the ad revenue rate of L-type advertisers β_I ;
- (iv) the advertising fee to L-type advertisers w_i^L , $i \in \{M, C\}$ increases with the ad revenue rate of L-type advertisers β_L .

Proposition 1 indicates that several factors, such as the revenue rate of H-type advertisers, influence the advertising fee decision of the social platform. In general, as the revenue rate of any type of advertisers increases, the social platform will adjust the advertising fee to both types of advertisers to realize the optimal profit. Intuitively, the optimal advertising fee to H-type (L-type) advertisers increases with β_H (β_L). This is because the platform sets a higher price to control the number of advertisers when they gain more revenue. The optimal advertising fee to H-type advertisers with β_L . The reason is that, when β_L increases, there will be more L-type advertisers on the social platform, which causes the reduction of consumer demand. As a result, H-type advertisers gain less revenue. Thus, the platform sets a lower advertising fee to retain H-type advertisers.

Interestingly, when β_H is sufficiently higher than β_L , the optimal advertising fee to L-type advertisers increases with β_H . The reason is that the social platform can make enough profit from H-type advertisers, and the entrance of L-type advertisers is ignored due to the small ad revenue that they generate. When β_L is moderately lower than β_H , the optimal advertising fee to L-type advertisers decreases with β_H . In this case, the profit from L-type advertisers is not so small that the platform needs to set a lower price to serve more L-type advertisers. In addition, when the difference between β_H and β_L is medium, *b* also influences the advertising fee to L-type advertisers. Specifically, the optimal advertising fee to L-type advertisers decreases with β_H when *b* is very high, and the optimal advertising fee to L-type advertisers increases with β_H when *b* is very low. A lower *b* means that the proportion of L-type advertisers is very high, and the social platform needs to control the number of L-type advertisers to ensure the proper number of advertisers on the platform. When *b* is very high, the social platform can reduce the advertising fee to retain more L-type advertisers. Based on the analysis above, the social platform can adjust the advertising fee to both types of advertisers when the revenue rate of any type advertisers changes.

Given the profits under the advertising and brokerage revenue models, we can answer the question: Which revenue model is optimal for the social platform? The following propositions summarize the answers to this question.

Proposition 2: Suppose that the social platform adopts the CPM ad revenue model: (i) when $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{MB1}$, the social platform always prefers the advertising model; (ii) when $s_{MB2} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$, the social platform always prefers the brokerage model;

(iii) when $s_{MB1} < s < s_{MB2}$, the social platform prefers the advertising model when $c \in (0, c_{MB})$, while the social platform prefers the brokerage model when $c \in (c_{MB}, 1)$,

$$where \ s_{MB1} = \frac{8(v-\gamma)(b\beta_{H}^{2}-b\beta_{L}^{2}+\beta_{L}^{2})-8(2v-2\gamma+\beta_{L})(v-\gamma+\beta_{L})+b(\beta_{H}-\beta_{L})(\beta_{H}-17\beta_{L}-24v+24\gamma)-9b^{2}(\beta_{H}-\beta_{L})^{2}}{8(v-\gamma)[8(2v-2\gamma+\beta_{L})(v-\gamma+\beta_{L})-b(\beta_{H}-\beta_{L})(\beta_{H}-17\beta_{L}-24v+24\gamma)+9b^{2}(\beta_{H}-\beta_{L})^{2}]} (v+v_{0})^{2}, \ s_{MB2} = \frac{b\beta_{H}^{2}-b\beta_{L}^{2}+\beta_{L}^{2}-2v+2\gamma}{16(v-\gamma)^{2}} (v+v_{0})^{2}, \ and \ c_{MB} \ is \ the \ unique \ solution \ of \ \frac{(v+v_{0})^{2}}{8(v-\gamma)} + s = \frac{4(v+v_{0})^{2}[c(b\beta_{H}-b\beta_{L}+\beta_{L})+v-\gamma][3c(b\beta_{H}-b\beta_{L}+\beta_{L})+4v-4\gamma]^{2}}{[8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}}.$$

Proposition 2 indicates that, when the social platform adopts the CPM ad revenue model under the advertising model, the optimal revenue model is determined by the fixed fee charged by the social platform to the seller *s* and the disutility of each advertiser to the consumers *c*. The revenue-model decision of the social platform is illustrated in Figure 1. As shown in region (i), when the fixed fee is very low (i.e., $s_{MB1} > 0, 0 < s < s_{MB1}$) or the subsidy that the platform needs to provide to retain the seller is very high (i.e., $s_{MB1} > 0, 0 < s < s_{MB1}$) or the subsidy that $0, -\frac{(v+v_0)^2}{8(v-\gamma)} < s < 0$ or $s_{MB1} < 0, -\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{MB1}$), it is difficult for the social platform to generate more revenue only by charging the transaction fee under the brokerage model. Hence, it is better for the social platform to make a profit by attracting

transaction fee under the brokerage model. Hence, it is better for the social platform to make a profit by attracting advertisers to join the platform, and the advertising model is always a better choice. As *s* further increases (i.e., $s_{MB1} < s < s_{MB2}$), the social platform makes more profits under the advertising

As s further increases (i.e., $s_{MB1} < s < s_{MB2}$), the social platform makes more profits under the advertising model when the disutility of each advertiser to the consumers is very low (i.e., $0 < c < c_{MB}$), and the social platform makes more profits under the brokerage model when the disutility of each advertiser to the consumers is very high (i.e., $c_{MB} < c < 1$). Specifically, as illustrated in region (ii), when *c* is very low, consumers feel little displeasure with advertisements on the platform. As a result, the number of consumers who are willing to consume on the social platform are large enough to attract sufficient advertisers to display their advertisements on the platform. Thus, the social platform makes more profits by charging an advertising fee to the advertisers and prefers the advertising model.

However, in region (iii) of Figure 1, where *c* exceeds the threshold c_{MB} , consumers are sensitive to advertisements, and they prefer to use the consuming service without any advertisements. Clearly, it is difficult for the social platform to attract advertisers through the commerce feature, and the social platform cannot gain profit

under the advertising model. Therefore, the brokerage model is a better choice. Further, when the fixed fee is very high (i.e., $s_{MB2} < s < \frac{(\nu + \nu_0)^2}{16(\nu - \gamma)}$), which corresponds to region (iv), the social platform benefits more under the brokerage model because a sufficiently high fixed fee guarantees that the platform's profit under the brokerage model always exceeds that under the advertising model.



Figure 1: Revenue-Model Decision of Social Platform under CPM Ad Revenue Model $(v = 0.9, v_0 = 0.2, \gamma = 0.1, \beta_H = 2.8, \beta_L = 1, \text{ and } b = 0.9)$

Proposition 3: Suppose that the social platform adopts the CPC ad revenue model: (i) when $s_{CB} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$, the social platform always prefers the brokerage model; (ii) when $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{CB}$, the social platform prefers the advertising model when $c \in (0, c_{CB})$, while the social $\begin{array}{l} platform \ prefers \ the \ brokerage \ model \ when \ c \in (c_{CB}, 1). \\ where \ \ s_{CB} = \frac{\tau^2 (b\beta_H^2 - b\beta_L^2 + \beta_L^2) - 2v + 2\gamma}{16(v - \gamma)^2} (v + v_0)^2 \ , \ and \ \ c_{CB} \ is \ the \ unique \ solution \ of \ \ \frac{(v + v_0)^2}{8(v - \gamma)} + s = \frac{\tau^2 (1 - c)^2 (v + v_0)^2 (b\beta_H^2 - b\beta_L^2 + \beta_L^2)}{8[2v - 2\gamma + c(1 - c)\tau\beta_L] [v - \gamma + c(1 - c)\tau(\beta_H - \beta_L)] c(1 - c)\tau(\beta_H - \beta_L) [c(1 - c)\tau(\beta_H - \beta_L) - 24v + 24\gamma] + 9b^2c^2\tau^2 (1 - c)^2(\beta_H - \beta_L)^2}. \end{array}$

Proposition 3 implies that, when the social platform adopts the CPC ad revenue model, the fixed fee charged by the social platform to the seller s and the disutility of each advertiser to the consumers c also have a significant impact on the social platform's revenue-model decision. The details are illustrated in Figure 2. As shown in region (i), when the fixed fee s and the disutility of advertisements to consumers c are both very low (i.e., $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < 1$ s_{CB} and $0 < c < c_{CB}$), consumers receive little negative effects from advertisements, and, thus, the social platform can attract a mass of advertisers and consumers who participate at the same time and makes more profits under the advertising model.

Further, we find that, in region (ii), where the fixed fee s is very low, although the disutility of advertisements to consumers c is very high (i.e., $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{CB}$ and $c_{CB} < c < 1$), consumers feel significantly averse to advertisers, which leads to the reduction of consumer demand. Hence, the social platform cannot attract enough advertisers through the commerce feature, and the social platform gains more profits under the brokerage model. In region (iii) of Figure 2, the sufficiently high fixed fee (i.e., $s_{CB} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$) ensures that the social platform always makes more profits under the brokerage model.

From the findings of Propositions 2 and 3, we can make some meaningful suggestions from a managerial perspective. Our results indicate that the social platform should pay attention to the fixed fee and the interaction between advertisers and consumers when choosing a revenue model. The social platform also can maintain a specific revenue model by adjusting the fixed fee and the negative effect caused by advertisers. For example, if the social platform is willing to make a profit through the advertising model, it should pay attention to the entrance of advertisers. To do so, the platform should comprehensively analyze consumers' demands and preferences through their browsing or operational behavior and achieve accurate advertising to control the disutility of advertisements to consumers.



Figure 2: Revenue-Model Decision of Social Platform under CPC Ad Revenue Model $(v = 0.9, v_0 = 0.2, \gamma = 0.1, \beta_H = 2.8, \beta_L = 1, b = 0.9, \text{ and } \tau = 0.9)$

We next investigate the influence of the characteristics of advertisers, such as the revenue rate of both types of advertisers, on the revenue-model decision of the social platform. The results are summarized in Propositions 4 and 5.

Proposition 4: Suppose that the social platform adopts the CPM ad revenue model, all other parameters being equal:

- (i) the social platform is more likely to choose the advertising model with an increase in the ad revenue rate of H-
- type advertisers β_H ; (ii) when $v > \frac{3bc\beta_H(\beta_H \beta_L)}{4\beta_L} + \gamma$, the social platform is more likely to choose the advertising model with an increase

in the ad revenue rate of L-type advertisers β_L , while when $v < \frac{3bc\beta_H(\beta_H - \beta_L)}{4\beta_L} + \gamma$, the social platform is more

likely to choose the brokerage model with an increase in the ad revenue rate of L-type advertisers β_L ; (iii) when $v > \frac{3c[b\beta_H - (1-b)\beta_L](\beta_H - \beta_L)}{4(\beta_H + \beta_L)} + \gamma$, the social platform is more likely to choose the advertising model with an increase in the proportion of H-type advertisers b, while when $v < \frac{3c[b\beta_H - (1-b)\beta_L](\beta_H - \beta_L)}{4(\beta_H + \beta_L)} + \gamma$, the social

platform is more likely to choose the brokerage model with an increase in the proportion of H-type advertisers b.

Proposition 5: Suppose that the social platform adopts the CPC ad revenue model, all other parameters being equal:

(i) the social platform is more likely to choose the advertising model with an increase in the ad revenue rate of Htype advertisers β_H ;

- (ii) when $v > \frac{3bc(1-c)\tau\beta_H(\beta_H-\beta_L)}{4\beta_L} + \gamma$, the social platform is more likely to choose the advertising model with an increase in the ad revenue rate of L-type advertisers β_L , while when $v < \frac{3bc(1-c)\tau\beta_H(\beta_H-\beta_L)}{4\beta_L} + \gamma$, the social platform is more likely to choose the brokerage model with an increase in the ad revenue rate of L-type
- $advertisers \ \beta_L;$ (iii) when $v > \frac{3c(1-c)\tau[b\beta_H (1-b)\beta_L](\beta_H \beta_L)}{4(\beta_H + \beta_L)} + \gamma$, the social platform is more likely to choose the advertising model $d_{\mu} = \frac{1}{2} \frac{1}{2}$

with an increase in the proportion of H-type advertisers b, while when $v < \frac{3c(1-c)\tau[b\beta_H - (1-b)\beta_L](\beta_H - \beta_L)}{4(\beta_H + \beta_L)} + \gamma$, the social platform is more likely to choose the brokerage model with an increase the proportion of H-type advertisers b.

Propositions 4 and 5 illustrate the influence of the characteristics of advertisers on the social platform's revenue-model decision when the advertising model is adopted by CPM and CPC models, respectively. The details are as follows. Proposition 4 (i) and Proposition 5 (i) reveal that the region where the social platform chooses the advertising model expands with the ad revenue rate of H-type advertisers. This is because an increase in the ad revenue rate of H-type advertisers leads to more H-type advertisers' joining the social platform. As a result, the social platform makes more profits under the advertising model when H-type advertisers gain more ad revenue.

As shown in Proposition 4 (ii) and Proposition 5 (ii), we can know that, when β_L is moderately lower than β_H , the social platform is more likely to choose the advertising model with an increase in β_L . When β_L is sufficiently lower than β_H , the social platform is more likely to choose the advertising model with an increase in β_L when v is very high; and the social platform is more likely to choose the brokerage model with an increase in β_L when v is very low. We can easily understand that an increase in β_L can attract more advertisers and then bring more profits to the social platform under the advertising model. When v is very low, consumers receive little utility from consuming on the social platform, and the disutility from advertisements matters more. In addition, because the difference between β_L and β_H is significantly large, β_L is so low that an increase in β_L brings little advertising revenue for the platform and causes great disutility to consumers. Consequently, the region where the social platform chooses the advertising model shrinks with β_L .

Corresponding to Propositions 4 (iii) and 5 (iii), when β_L is moderately lower than β_H , the social platform is more likely to choose the advertising model with an increase in b. When β_L is significantly lower than β_H , however, the social platform is more likely to choose the advertising model with an increase in b when v is very high; and the social platform is more likely to choose the brokerage model with an increase in b when v is very low. The intuition is as follows. Compared with L-type advertisers, H-type advertisers generate more profits from an advertisement, and they are more likely to participate in the platform; thus, the social platform generates profit mainly from H-type advertisers. As a result, as the proportion of H-type advertisers increases, the total number of advertisers on the platform will increase, and the social platform will obtain more profits from the ad service. Interestingly, when β_L is significantly lower than β_H , and v is very low, it is difficult for consumers to receive utility from the commerce feature' thus, the disutility caused by advertisers matters more, and both the optimal consumer demand and the number of advertisers decrease with b. Consequently, the social platform is more likely to choose the brokerage model.

In summary, Propositions 4 and 5 suggest that, when the social platform chooses the revenue model for a commerce feature, except for the fixed fee and the interaction between advertisers and consumers discussed above, it also should pay attention to the characteristics of advertisers, such as the revenue rate of different advertisers and the composition of advertisers, by controlling the entrance of advertisers. Further, the social platform can adjust the value of commerce feature by adopting advanced technology to make a better revenue-model choice.

Comparison of Customer Surplus and Profit Allocation in Supply Chain 5.

In this section, we analyze the comparison of customer surplus (CS) and profit allocation in the supply chain between the two different revenue models. Propositions 6 and 7 show the results of the comparison of CS under the CPM and CPC ad revenue models, respectively. The values for CS in each revenue model $CS_i, i \in \{M, C, B\}$ are shown in Appendices.

Proposition 6: Define a threshold value $v_1 = \frac{\sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{4}$. Suppose that the social platform adopts the CPM ad revenue model:

(i)when $v > v_1 + \gamma$, CS is higher under the advertising model:

(ii) when $\gamma < v < v_1 + \gamma$, there exists a threshold $c_0 \in (0,1)$, CS is higher under the advertising model when $c \in (0, c_0)$, while CS is higher under the brokerage model when $c \in (c_0, 1)$.

Proposition 6 compares the CS between CPM advertising and brokerage models. The details are illustrated in Figure 3. We can know that, when v is very high (i.e., $v > v_1 + \gamma$), CS is always higher under the advertising model. This is because the seller could set a lower price under the advertising model in which he or she pays nothing for selling on the social platform. In addition, the higher v ensures that the disutility caused by advertisers has little influence on consumers.

When v is very low (i.e., $\gamma < v < v_1 + \gamma$), however, the comparison of CS depends on c. Specifically, when the advertising model is adopted, a lower c ($0 < c < c_0$) brings less disutility to consumers which leads CS to be higher. However, when c is very high (i.e., $c_0 < c < 1$), consumers experience great displeasure from advertisements; thus, CS is higher under the brokerage model.



Proposition 7: Define a threshold value $v_2 = \frac{\tau \sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{16}$. Suppose that the social platform adopts the CPC ad revenue model:

(i) when $v > v_2 + \gamma$, CS is higher under the advertising model;

(ii) when $\gamma < v < v_2 + \gamma$, there exists two thresholds $c_1 \in (0, \frac{1}{2})$ and $c_2 \in (\frac{1}{2}, 1)$, CS is higher under the advertising model when $c \in (0, c_1)$ or $c \in (c_2, 1)$, while CS is higher under the brokerage model when $c \in (c_1, c_2)$.

Proposition 7 and Figure 4 show the comparison of CS between the CPC advertising model and brokerage model. Similar to Proposition 6, when v is very high (i.e., $v > v_2 + \gamma$), CS under the advertising model is always higher than that under the brokerage model. When v is very low (i.e., $\gamma < v < v_2 + \gamma$), however, the comparison of CS also depends on the value of c.



 $(v_0 = 0.2, \gamma = 0.1, \beta_H = 2.8, \beta_L = 1, b = 0.9, \text{ and } \tau = 0.9)$

Interestingly, when c is very low or very high (i.e., $0 < c < c_1$ or $c_2 < c < 1$), CS under the advertising model is higher. We can easily understand the situation in which c is very low; thus, we focus on explaining the situation in which c is very high. The sufficiently high c leads to large number of consumers' leaving the platform, and the social platform will control the number of advertisers by promoting an advertising fee to retain consumers. Although c is high, the smaller number of advertisers ensures that CS is higher under the advertising model. Further, when $c_1 < c < c_2$, c is moderately high and the number of advertisers is not too small; thus, CS is higher under the brokerage model.

One may expect the effect of revenue-model decision on the profit allocation in supply chain, and we provide the result in Proposition 8.

Proposition 8:

(1) Suppose that the social platform adopts the CPM ad revenue model:

- (i) when $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{M1}$, the social platform is allocated more profits in the supply chain under the advertising model;
- (ii) when $s_{M2} < s < \frac{(v+v_0)^2}{16(v-v)}$, the social platform is allocated more profits in the supply chain under the brokerage model:
- (iii) when $s_{M1} < s < s_{M2}$, the social platform is allocated more profits in the supply chain under the advertising model when $c \in (0, c_M)$, while the social platform is allocated more profits in the supply chain under the brokerage model when $c \in (c_M, 1)$,

 $\text{where} \quad s_{M1} = \left[\frac{(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})[8(2v - 2\gamma + \beta_{L})(v - \gamma + \beta_{L}) - b(\beta_{H} - \beta_{L})(\beta_{H} - 17\beta_{L} - 24v + 24\gamma) + 9b^{2}(\beta_{H} - \beta_{L})^{2}]}{(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})[8(2v - 2\gamma + \beta_{L})(v - \gamma + \beta_{L}) - b(\beta_{H} - \beta_{L})(\beta_{H} - 17\beta_{L} - 24v + 24\gamma) + 9b^{2}(\beta_{H} - \beta_{L})^{2}] + 4(\overline{\beta} + v - \gamma)(3\overline{\beta} + 4v - 4\gamma)^{2}} - \frac{2}{3} \frac{3(v + v_{0})^{2}}{16(v - \gamma)}, \quad s_{M2} = \frac{3(v + v_{0})^{2}}{16(v - \gamma)} \left[\frac{b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2}}{b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2} + 4v - 4\gamma} - \frac{2}{3} \right], \quad and \quad c_{M} \quad is \quad the \quad unique \quad solution \quad of \quad \frac{2(v + v_{0})^{2} + 16s(v - \gamma)}{3(v + v_{0})^{2}} = \frac{16(v - \gamma)}{3(v + v_{0})^{2}} = \frac{16(v - \gamma)}{3(v + v_{0})^{2}} + \frac{16(v - \gamma)}{3(v + v_{0})^{2}} = \frac$ $\left(b\beta_{H}^{2}-b\beta_{L}^{2}+\beta_{L}^{2}\right)\left[8(2\nu-2\gamma+\beta_{L})(\nu-\gamma+\beta_{L})-b(\beta_{H}-\beta_{L})(\beta_{H}-17\beta_{L}-24\nu+24\gamma)+9b^{2}(\beta_{H}-\beta_{L})^{2}\right]$

- $\overline{(b\beta_{H}^{2}-b\beta_{L}^{2}+\beta_{L}^{2})[8(2v-2\gamma+\beta_{L})(v-\gamma+\beta_{L})-b(\beta_{H}-\beta_{L})(\beta_{H}-17\beta_{L}-24v+24\gamma)+9b^{2}(\beta_{H}-\beta_{L})^{2}]+4(\overline{\beta}+v-\gamma)(3\overline{\beta}+4v-4\gamma)^{2}}$ (2) Suppose that the social platform adopts the CPC ad revenue model:
- (i) when $s_c < s < \frac{(\nu+\nu_0)^2}{16(\nu-\nu)}$, the social platform is allocated more profits in the supply chain under the brokerage model;
- (ii) when $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_c$, the social platform is allocated more profits in the supply chain under the advertising model when $c \in (0, c_c)$, while the social platform is allocated more profits in the supply chain under the brokerage model when $c \in (c_c, 1)$,

$$\frac{where}{16(v-\gamma)} \frac{s_{C} = \frac{3(v+v_{0})^{2}}{16(v-\gamma)} \left[\frac{\tau^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})}{\tau^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2}) + 4v} - \frac{2}{3} \right], and c_{C} is the unique solution of \frac{2(v+v_{0})^{2} + 16s(v-\gamma)}{3(v+v_{0})^{2}} = \frac{1}{4[c(1-c)\tau\overline{\beta} + v-\gamma][3c(1-c)\tau\overline{\beta} + 4v - 4\gamma]^{2}}$$

 $1 + \frac{4[c(1-c)\tau\rho + v - \gamma_{J}]sc(1-c)\tau\rho + 4v - 4\gamma_{J}^{-}}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})[8[2v - 2\gamma + c(1-c)\tau\beta_{L}][v - \gamma + c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H} - \beta_{L})[c(1-c)\tau\beta_{H} - 17c(1-c)\tau\beta_{L} - 24v + 24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H} - \beta_{L})^{2}}]$

Proposition 8 shows the supply chain profit allocation under different revenue models. We use parameter λ_i ($\lambda_i = \frac{\pi_i}{\pi_i + \pi_i}$, $i \in \{M, C, B\}$) to characterize the allocation of total profit between the social platform and the seller [Fiala 2015]. Proposition 8 (1) illustrates the comparison of profit allocation between the CPM advertising model and brokerage model. When the fixed fee is very high (i.e., $s_{M2} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$), the fixed fee is sufficiently high for the social platform to be allocated more profits under the brokerage model. When the fixed fee is very low (i.e., $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{M1}$), the social platform receive less profit from the seller, and, thus, the social platform is allocated more profits under the advertising model. When the fixed fee is medium (i.e., $s_{M1} < s < s_{M2}$), the profit allocation also depends on the value of *c*. A lower *c* can attract more consumers to join the platform and then advertising model. In contrast, a higher *c* ensures that the platform could be allocated more profits under the brokerage model.

Proposition 8(2) shows the comparison of profit allocation between the CPC advertising model and the brokerage model. The adoption of the CPC ad revenue model leads to less revenue under the advertising model for the platform; thus, the situation in which platform could always be allocated more profits under advertising model does not appear. Further, the other analysis is similar to Proposition 8(1). Compared with the results in the revenue-model decision, we find that the parameter condition under which each revenue model is optimal also enables the social platform to be allocated more profits from the supply chain.

6. Extended Model with Non-linear Disutility of Advertisers

In the basic model, the disutility of advertisers to the consumers is increasing linearly. In reality, the impact of an additional advertiser on the disutility perceived by any consumer increases in the total number of advertisers. Hence, we explore a generalization of our model by considering a non-linear disutility of advertisers to consumers in this section. In the basic model, consumers derive a disutility cQ_i , $i \in \{M, C\}$ when advertisers display their content on the social platform, where *c* represents the aversion of a consumer from the participation of each advertiser, and Q_i , $i \in \{M, C\}$ denotes the number of advertisers on the social platform. In this section, we consider that consumers derive a disutility $cQ_i + eQ_i^2$, $i \in \{M, C\}$ when advertisers display their content on the social platform, we consider that enables the disutility of the advertiser to the consumer to increase concavely. The analyses in Sections 4 and 5 correspond to the linear disutility of advertisers to consumers (where e = 0). Hence, consumer utility under the advertising model can be expressed as $u_i = v_0 + \theta v + \gamma D_i - (cQ_i + eQ_i^2) - p_i$, $i \in \{M, C\}$.



Figure 5: Revenue-Model Decision of Social Platform under CPM Ad Revenue Model $(v = 0.9, v_0 = 0.2, \gamma = 0.1, \beta_H = 2.8, \beta_L = 1, b = 0.9, \text{ and } e = 0.01)$



Figure 6: Revenue-Model Decision of Social Platform under CPC Ad Revenue Model $(v = 0.9, v_0 = 0.2, \gamma = 0.1, \beta_H = 2.8, \beta_L = 1, b = 0.9, \tau = 0.99, \text{ and } e = 0.01)$

In this case, the social platform's profit maximization problem is analytically intractable. Hence, we employ numerical analysis to explore the revenue-model decision of the social platform. We conduct a numerical analysis for a wide range of parameter values, with the following criteria used for choosing the parameter values. The revenue rate of both types of advertisers is sufficiently high to ensure that advertisers are willing to advertise on the social platform; the adjustment parameter e is sufficiently low to guarantee that consumers are willing to consume on the platform; and the value of other parameters are randomly selected in the value range.

From Figures 5 and 6, we find that, when the fixed fee s and the disutility of advertisements to consumers c are both very low, the social platform can attract a mass of advertisers and consumers who participate at the same time and make more profits under the advertising model. Further, when the fixed fee s is very low, although the disutility of advertisements to consumers c is very high, the social platform cannot attract enough advertisers through the commerce feature, and it gains more profits under the brokerage model. When the fixed fee s is very high, the social platform always makes more profits under the brokerage model. Overall, the results remain robust when the disutility of advertisers to consumers increases concavely.

Compared with the basic model, we find that the social platform is more likely to choose the brokerage model when the disutility of advertisers to consumers increases concavely. The reason is as follows. When gaining another advertiser has a large impact on the disutility perceived by all consumers, consumers prefer to use the consuming service without any advertisement, and it is difficult for the social platform to attract advertisers by the commerce feature. Thus, the social platform makes more profits under the brokerage model. We can make some meaningful suggestions from a managerial perspective. The social platform should adopt the brokerage model when the disutility of advertisers to consumers increases concavely. If the social platform insists on adopting the advertising model, it should reduce advertising or achieve target advertising by analyzing the data collected from consumers.

7. Conclusion

The rapid development of social media and the wide application of e-commerce has inspired social commerce, and social platforms increasingly integrate the commerce feature into their services. Anecdotal observations, however, suggest that different social platforms may deploy different revenue models and adopt mainly the advertising model or the brokerage model. Moreover, the social platform can choose CPM and CPC ad revenue models for its ad service under the advertising model. Thus, we wondered what the optimal revenue model is for the social platform to profit from the commerce feature. This study develops an analytical model to address this question. We first examine the optimal revenue model for the social platform and then analyze several factors, such as the ad revenue rate of both types of advertisers, on the revenue-model decision and the advertising-fee decision. We further compare the consumer surplus and profit allocation in the supply chain between two revenue models. The key findings and managerial implications are as follows.

First, when the fixed fee is very low, the advertising model is always an optimal choice. When the fixed fee is medium, the social platform obtains more profits under the advertising model if the disutility of advertisements is very low; otherwise, the social platform obtains more profits under the brokerage model. When the fixed fee is very high, the social platform benefits more under the brokerage model. Interestingly, the adoption of CPC ad revenue model leads to less revenue under the advertising model for the platform; thus, the situation in which the advertising model is always an optimal choice does not appear. This result provides a significant managerial implication for a social platform that intends to integrate the commerce feature. It implies that the social platform should pay attention to the fixed fee and the interaction between advertisers and consumers when choosing the revenue model. The social platform also can maintain a specific revenue model by adjusting the fixed fee and the negative effect caused by advertisements.

Second, given the change of the ad revenue rate of advertisers, the social platform may make different revenue model decisions, and adjust the advertising fee under the advertising model, to realize their optimal profit. In particular, all other parameters' being equal, as the revenue rate of H-type advertisers increases, the platform prefers the advertising model, and the optimal advertising fee to H-type advertisers increases while the optimal advertising fee to L-type advertisers decreases. Similarly, the optimal advertising fee to L-type advertisers increases with the revenue rate of L-type advertisers. The change in the revenue-model decision also depends on the value of the commerce feature. Interestingly, as the revenue rate of H-type advertisers increases, the advertising fee to L-type advertisers depends on the difference of revenue rate between H-type advertisers and L-type advertisers. This suggests that, when the social platform chooses the revenue model for the commerce feature, it also should pay attention to the characteristics of advertisers. Alternatively, the social platform can adjust the value of the commerce feature to make a better revenue-model choice. Further, the social platform can adjust the advertising fee to both types of advertisers when the revenue rate of any type of advertiser changes.

Third, when the value of commerce feature is very high, the customer surplus is higher under the advertising model, and when the value of commerce feature is very low, the customer surplus is higher under the advertising model when the disutility of advertisements is very low; otherwise, the customer surplus is higher under the brokerage model. Interestingly, when the social platform adopts the CPC advertising model, the customer surplus is higher under the disutility of advertisements is very low; of advertisements is very high. This result also provides a significant managerial implication for the social platform. To guarantee a higher customer surplus, the social platform can adjust the value of the commerce feature by adopting advanced technology or control the disutility of advertisements by achieving accurate advertising.

Finally, when the fixed fee is very low, the social platform could be allocated more profits from the supply chain under the advertising model. When the fixed fee is medium, the social platform could be allocated more profits under the advertising model if the disutility of advertisements is low; otherwise, the social platform could be allocated more profits under the brokerage model. When the fixed fee is very high, the social platform could be allocated more profits under the brokerage model. Interestingly, the adoption of the CPC ad revenue model leads to less revenue under the advertising model for the platform; thus, the situation in which the platform could always be allocated more profits under the advertising model does not appear. This result implies that the social platform should pay attention to the fixed fee and the interaction between advertisers and consumers if it is willing to be allocated more profits from the supply chain.

8. Future Research

In this section, we present several limitations of our current research and put forward suggestions for future research. First, we consider the situation in which the social platform chooses only one of the revenue models and ignore that a social platform can adopt both revenue models simultaneously to generate more profits. For future research, examining a hybrid strategy is a possible but challenging direction. Second, we disregard the competition among sellers. It would be of interest to consider several sellers who are competing on the social platform and take their popularity into account. Third, different advertising spaces on the social platform will generate different advertising fees, and we ignore the situation in which advertisers compete for better advertising space through bidding [Chen & Stallaert 2014; Xu et al. 2011]. Thus, the analysis can be further extended to account for competition between advertisers. Fourth, we consider a monopoly market for a social platform and disregard the competition between social platforms. It would be valuable to investigate the game between two social platforms. Future research can consider a competitive market in which two social platforms compete for consumers and deploy either one of the two revenue models. Finally, we consider only the situation in which the revenue rates of different advertisers are exogenous variables and ignore the game between the social platforms.

research could consider the endogeneity of the advertisers' revenue rate and the game between the platforms and advertisers.¹

Acknowledgments

We thank the review team for detailed and constructive comments that greatly improved this paper. We gratefully acknowledge the financial support from the Key Program of National Natural Science Foundation of China (No.71631003). Zhiyong Li also acknowledges financial support from the National Science Foundation of China (No.71772167).

REFERENCES

- Akman, I., and A. Mishra, "Factors Influencing Consumer Intention in Social Commerce Adoption," *Information Technology & People*, Vol. 30, No. 2:356-370, 2017.
- Armstrong, M., "Competition in Two-sided Markets," *The RAND Journal of Economics*, Vol. 37, No. 3:668-691, 2006.
- Busalim, A.H., and A.R. Hussin, "Understanding Social Commerce: A Systematic Literature Review and Directions for Future Research," *International Journal of Information Management*, Vol. 36:1075-1088, 2016.
- Chen, J.Q., and J. Stallaert, "An Economic Analysis of Online Advertising Using Behavioral Targeting," MIS *Quarterly*, Vol. 38, No. 2:429-449, 2014.
- Chen, J.Q., M. Fan, and M.Z. Li, "Advertising versus Brokerage Model for Online Trading Platforms," *MIS Quarterly*, Vol. 40, No. 3:575-596, 2016.
- Chen, J.Q., and Z.L. Guo, "New Media Advertising and Retail Platform Openness," Social Science Electronic Publishing, Vol. 21, 2015.
- Constine, J., "Facebook Tests Buy Button to Let You Purchase Stuff without Leaving Facebook," 2014. Retrieved from https://techcrunch.com/2014/07/17/facebook-buybutton/.
- Dou G.W., X.D. Lin, R. Chi, and Z.X. Zheng, "Pricing Strategy of a Two-sided Platform under Consumer Categorization", *Journal of Electronic Commerce Research*, Vol. 21, No. 2:253-272, 2020.
- Ellison, G., and D. Fudenberg, "The Neo-Luddite's Lament: Excessive Upgrades in the Software Industry," *The RAND Journal of Economics*, Vol. 31, No. 2:253-272, 2000.
- Etzion, H., and M.S. Pang, "Complementary Online Services in Competitive Markets: Maintaining Profitability in the Presence of Network Effects," *MIS Quarterly*, Vol. 38, No. 1:231-248, 2014.
- Feiner, L., "Pinterest Launches a Small Business Shop Ahead of the Holidays," 2019. Retrieved from https://www.cnbc.com/2019/11/25/pinterest-launches-a-small-business-shop-ahead-of-the-holidays.html.
- Fiala, P., "Profit Allocation Games in Supply Chains," *Central European Journal of Operations Research*, Vol. 24:267-281, 2016.
- Gesenhues, A., "Facebook Expanding Marketplace Ads to More Countries & Campaign Objectives," 2018. Retrieved from https://marketingland.com/facebook-expanding-marketplace-ads-to-more-countries-campaign-objectives-241455?utm_source=tuicool&utm_medium=referral.
- Hu, X., Q. Huang, X.P. Zhong, R.M. Davison, and D.T. Zhao, "The Influence of Peer Characteristics and Technical Features of a Social Shopping Website on a Consumer's Purchase Intention," *International Journal of Information Management*, Vol. 36:1218-1230, 2016.
- Jiang, C.Q., R.M. Rashid, and J.F. Wang, "Investigating the Role of Social Presence Dimensions and Information Support on Consumers' Trust and Shopping Intentions," *Journal of Retailing and Consumer Services*, Vol. 51:263-270, 2019.
- Jing, B., "Network Externalities and Market Segmentation in a Monopoly," *Economic Letters*, Vol. 95, No. 1:7-13, 2007.
- Jing, X.Q., and J.H. Xie, "Group Buying: A New Mechanism for Selling Through Social Interactions," *Management Science*, Vol. 57, No. 8:1354-1372, 2011.
- Kumar, V., V. Bhaskaran, R. Mirchandani, and M. Shah, "Practice Prize Winner—Creating a Measurable Social Media Marketing Strategy: Increasing the Value and ROI of Intangibles and Tangibles for Hokey Pokey," *Marketing Science*, Vol. 32, No. 2: 194-212, 2013.
- Lee, R., "Facebook Wants You to Buy and Sell Your Stuff in Their Marketplace," 2018. Retrieved from https://www.soyacincau.com/2018/04/27/facebook-wants-you-to-buy-and-sell-your-stuff-in-their-marketplace/.
- Li, C.Y., "How Social Commerce Constructs Influence Customers' Social Shopping Intention? An Empirical Study of a Social Commerce Website," *Technological Forecasting & Social Change*, Vol. 144:282-294, 2019.

¹ We thank an anonymous reviewer for this suggestion.

- Li, S.L., H.K. Cheng, and Y. Jin, "Optimal Distribution Strategy for Enterprise Software: Retail, SaaS, or Dual Channel?" *Production and Operations Management*, Vol. 27, No. 11:1928-1939, 2018.
- Li, X., and Y. Chen, "Corporate IT Standardization: Product Compatibility, Exclusive Purchase Commitment, and Competition Effects," *Information Systems Research*, Vol. 23, No. 4:1158-1174, 2012.
- Li, Z.Y., G.F. Nan, and M.Q. Li, "Advertising or Freemium: The Impacts of Social Effects and Service Quality on Competing Platforms," *IEEE Transactions on Engineering Management*, Vol. 67, No. 1:220-233, 2020.
- Lin, M, X.Q. Ke, and A.B. Whinston, "Vertical Differentiation and a Comparison of Online Advertising Models," *Journal of Management Information Systems*, Vol. 29, No. 1:195-235, 2012.
- Lopez, F.J., Y.C. Li, W. Su, and C.Y. Feng, "To Have or Have Not: Buy Buttons on Social Platforms," *Journal of Business Research*, Vol. 105:33-48, 2019.
- Ma, L.Y., R. Krishnan, and A.L. Montgomery, "Latent Homophily or Social Influence? An Empirical Analysis of Purchase within a Social Network," *Management Science*, Vol. 61, No. 2:454-473, 2015.
- Nan, G.F., J.R. Yang, and R.L. Dou, "Do Only Review Characteristics Affect Consumers' Online Behaviors? A Study of Relationship Between Reviews," *Journal of Electronic Commerce Research*, Vol. 18, No. 4:330-345, 2017.
- Norris, M., "Little Red Book Shows Big User Numbers Don't Mean Big Profits," 2019. Retrieved from https://technode.com/2019/12/25/little-red-book-shows-big-users-dont-mean-bigprofits/?utm source=tuicool&ut m medium=referral.
- Perez, S., "Facebook Marketplace Expands into Home Services," 2018. Retrieved from https://techcrunch.com/2018/05/23/facebook-marketplace-expands-into-home-services/.
- Qiu, J.T., Y.H. Li, and Z.X. Lin, "Detecting Social Commerce: An Empirical Analysis on Yelp," Journal of Electronic Commerce Research, Vol. 21, No. 3:168-179, 2020.
- Rezaeian, A., S. Shokouhyar, and S. Yousefi, "Intention to Purchase Behavior on Social E-commerce Website across Cultures (Case Study: Iranian Online Purchaser)," *International Journal of Computers & Technology*, Vol. 15, No. 9:7077-7089, 2016.
- Ryan, J.K., D. Sun, and X.Y. Zhao, "Competition and Coordination in Online Marketplaces," *Production and Operations Management*, Vol. 21, No. 6:997-1014, 2012.
- Schiff, A., "Open and Closed Systems of Two-sided Networks," *Information Economics and Policy*, Vol. 15, No. 4:425-442, 2003.
- Wang, X., B. Baesens, and Z. Zhu, "On the Optimal Marketing Aggressiveness Level of C2C Sellers in Social Media: Evidence from China," Omega, Vol. 85:83-93, 2019.
- Wang, Y.G., S. Ma, and D.H. Li, "The Role of Regulatory Fit in Virtual Brand Communities," Journal of Electronic Commerce Research, Vol. 18, No. 2:124-137, 2017.
- Xiang, L., X.B. Zheng, M.K. Lee, and D.T. Zhao, "Exploring Consumers' Impulse Buying Behavior on Social Commerce Platform: The Role of Parasocial Interaction," *International Journal of Information Management*, Vol. 36, No. 3:333-347, 2016.
- Xu, L.Z., J.Q. Chen, and A.B. Whinston, "Price Competition and Endogenous Valuation in Search Advertising," *Journal of Marketing Research*, Vol. 48, No. 3: 566-586, 2011.
- Yahia, I.B., N.A. Neama, and L. Kerbache, "Investigating the Drivers for Social Commerce in Social Media Platforms: Importance of Trust, Social Support, and the Platform Perceived Usage," *Journal of Retailing and Consumer Services*, Vol. 41:11-19, 2018.
- Yang, Y.C., H.K. Cheng, C. Ding, and S.L. Li, "To Join or Not to Join Group Purchasing Organization: A Vendor's Decision," *European Journal of Operational Research*, Vol. 258, No. 2:581-589, 2017.
- Ye, Q., M. Xu, M. Kiang, and W.F. Wu, "In-depth Analysis of the Seller Reputation and Price Premium Relationship: A Comparison between Ebay US and Taobao China," *Journal of Electronic Commerce Research*, Vol. 14, No. 1:1-10, 2013.
- Zhang, P., and C.N. Wang, "The Evolution of Social Commerce: An Examination from the People, Management, Technology, and Information Dimensions," *Communications of the Association for Information Systems*, Vol. 31, No. 5:105-127, 2012.
- Zhou, L., P. Zhang, and H. Zimmermann, "Social Commerce Research: An Integrated View," *Electronic Commerce Research and Applications*, Vol. 12, No. 2: 61–68, 2013.

Appendix A: Value Ranges of Parameters

Assumption 1 gives the required conditions on the parameters values under which the platform has positive consumers and/or advertisers demand, and the consumer and/or advertiser market is fully covered.

- Assumption 1:
- (1) Under the CPM advertising model, the conditions are (i) $\beta_H 3\beta_L < 0$, (ii) $\nu > \gamma$, (iii) $0 < \beta_H < \frac{4(\nu - \gamma)}{\nu + \nu_0}$, (iv) $0 < \beta_L < \frac{4(\nu - \gamma)}{\nu + \nu_0}$.
- (2) Under the CPC advertising model, the conditions are (i) $\beta_H 3\beta_L < 0$, (ii) $\nu > \gamma$, (iii) $0 < \beta_H < \frac{4(\nu-\gamma)}{(\nu+\nu_0)(1-c)\tau}$, (iv) $0 < \beta_L < \frac{4(\nu-\gamma)}{(\nu+\nu_0)(1-c)\tau}$.
- (3) Under the brokerage model, the conditions are (i) $v > \gamma$, (ii) $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$

From the perspective of ensuring the positive net utility of the indifferent consumer and/or advertiser, the conditions (1)-(i)/(ii) and (2)-(i)/(ii) ensure the positive utility of the marginal consumer when the platform adopts advertising model. The conditions (1)-(i)/(ii) and (2)-(i)/(ii) ensure the positive utility of the marginal advertiser when the platform adopts advertising model. Similarly, the condition (3)-(i) ensures the positive utilities of the marginal consumers when the platform adopts the brokerage model.

From the view of guaranteeing the positive demand of each platform's consumer and/or advertiser base, the conditions (1)-(ii)/(iii) and (2)-(ii)/(iii) ensure that an optimal solution in which the platform has a positive demand of H-type advertisers prevails when the platform adopts an advertising model. Similarly, the conditions (1)-(ii)/(iv) and (2)-(ii)/(iv) ensure that an optimal solution in which the platform has a positive demand of L-type advertisers prevails when the platform adopts an advertiser has a positive demand of L-type advertisers prevails when the platform adopts an advertising model.

From the view of guaranteeing the positive revenue of the platform and seller, the condition (3)-(ii) ensures the positive profit of the platform and seller when the platform adopts the brokerage model. Finally, given the conditions (1)-(i)/(ii) and (2)-(i)/(ii), all the second order conditions (SOCs) in the main text are satisfied.

Appendix B: Proof of Lemma 1

A consumer will buy the product on the social platform when $u_M \ge 0$. We denote θ_M as the marginal consumers who are indifferent about consuming on the social platform, where $0 < \theta_M < 1$. Thus, we can derive the consumer demand under CPM advertising model as $D_M = 1 - \theta_M$. We further denote t_M^k , $k \in \{H, L\}$ as the fixed cost of a platform's marginal *k*-type advertisers who are indifferent about participating. By setting the advertising profit function as $U_M^k = 0, k \in \{H, L\}$, we get the location of the marginal advertisers who are indifferent about joining the social platform, $t_M^k = \beta_k D_M - w_M^k$, $k \in \{H, L\}$, $0 < t_M^k < 1$. Thus, $Q_M = Q_M^H + Q_M^L = b\beta_H D_M - bw_M^H + (1 - b)\beta_L D_M - (1 - b)w_M^L$. Hence, to realize the optimal results, the social platform announces its advertising fee w_M^L , $k \in \{H, L\}$, and the seller decides on the price of the products p_M to maximize their respective profits. Finally, we derive the optimal results as follows.

$$\begin{split} w_{M}^{H^{*}} &= \frac{(v+v_{0})[4(v-\gamma)\beta_{H}+3c(1-b)\beta_{L}\beta_{H}+4cb\beta_{H}^{2}+c(1-b)\beta_{L}^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}, \\ w_{M}^{L^{*}} &= \frac{(v+v_{0})[4(v-\gamma)\beta_{L}+3cb\beta_{L}\beta_{H}+4c(1-b)\beta_{L}^{2}+cb\beta_{H}^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}, \\ p_{M}^{*} &= \frac{2(v+v_{0})(c\bar{\beta}+v-\gamma)(3c\bar{\beta}+4v-4\gamma)}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}, \\ \pi_{M}^{*} &= \frac{4(v+v_{0})^{2}(c\bar{\beta}+v-\gamma)(3c\bar{\beta}+4v-4\gamma)^{2}}{[8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}}, \\ \Pi_{M}^{*} &= \frac{(v+v_{0})^{2}(b\beta_{H}^{-2}-b\beta_{L}^{-2}+\beta_{L}^{-2})}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]}, \\ Q_{M}^{*} &= \frac{(v+v_{0})[2\beta_{L}(2v-2\gamma+c\beta_{L})-b(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]}, \\ D_{M}^{*} &= \frac{2(v+v_{0})[2\beta_{L}(2v-2\gamma+c\beta_{L})-b(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]}{8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}, \\ D_{M}^{*} &= \frac{2(v+v_{0})[2\beta_{L}(2v-2\gamma+c\beta_{L})-b(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}}, \\ 0_{M}^{*} &$$

Appendix C: Proof of Lemma 2

A consumer will buy the product on the social platform when $u_c \ge 0$. We denote θ_c as the marginal consumers who are indifferent about consuming on the social platform, where $0 < \theta_c < 1$. Thus, we can derive the consumer demand under CPC advertising model as $D_c = 1 - \theta_c$. We further denote $t_c^k, k \in \{H, L\}$ as the fixed cost of a platform's marginal *k*-type advertisers who are indifferent about participating. By setting the advertising profit function as $U_c^k = 0, k \in \{H, L\}$, we get the location of the marginal advertisers who are indifferent about joining the social platform, $t_c^k = \beta_k D_c - w_c^k, k \in \{H, L\}$, $0 < t_c^k < 1$. Thus, $Q_c = Q_c^H + Q_c^L = b\beta_H D_c - bw_c^H + (1 - b)\beta_L D_c - (1 - b)w_c^L$. Hence, to realize the optimal results, the social platform announces its advertising fee w_c^L , $k \in \{H, L\}$, and the seller decides on the price of the products p_c to maximize their respective profits. Finally, we derive the optimal results as follows.

$$\begin{split} w_{C}^{H^*} &= \frac{(1-c)\tau(v+v_0)[4(v-\gamma)\beta_H + 3c(1-c)\tau(1-b)\beta_L\beta_H + 4c(1-c)\tau b\beta_H^2 + c(1-c)\tau(1-b)\beta_L^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2}, \\ w_{C}^{L^*} &= \frac{(1-c)\tau(v+v_0)[4(v-\gamma)\beta_L + 3c(1-c)\tau b\beta_L\beta_H + 4c(1-c)\tau(1-b)\beta_L^2 + c(1-c)\tau b\beta_H^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2}, \\ p_{C}^* &= \frac{2(v+v_0)[c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2}, \\ \pi_{C}^* &= \frac{4(v+v_0)^2[c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}, \\ p_{C}^* &= \frac{\tau^2(1-c)^2(v+v_0)^2(b\beta_H^2 - b\beta_L^2 + \beta_L^2)}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}, \\ p_{C}^* &= \frac{(1-c)\tau(v+v_0)\{2\beta_L[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}, \\ p_{C}^* &= \frac{(1-c)\tau(v+v_0)\{2\beta_L[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}, \\ p_{C}^* &= \frac{(1-c)\tau(v+v_0)\{2\beta_L[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}, \\ p_{C}^* &= \frac{(1-c)\tau(v+v_0)\{2\beta_L[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H - 17c(1-c)\tau\beta_L - 24v+24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2]}{8[2v-2\gamma+c(1-c)\tau\beta_L] - bc(1-c)\tau\beta_L - bc(1-c)\tau(\beta_H-\beta_$$

Appendix D: Proof of Lemma 3

A consumer will buy the product on the social platform when $u_B \ge 0$. We denote θ_B as the marginal consumers who are indifferent about consuming on the social platform, where $0 < \theta_B < 1$. Thus, we can derive the consumer demand under brokerage model as $D_B = 1 - \theta_B$. Thus, to realize the optimal results, the social platform announces its transaction fee *f* and the seller decides on his price for the products p_B to maximize their respective profits. Finally, we derive the optimal results as follows.

$$f^* = \frac{v + v_0}{2},$$

$$p_B^* = \frac{3v + 3v_0}{4},$$

$$\pi_B^* = \frac{(v + v_0)^2}{16v - 16\gamma} - s,$$

$$\Pi_B^* = \frac{(v + v_0)^2}{8v - 8\gamma} + s,$$

$$D_B^* = \frac{v + v_0}{4v - 4\gamma}.$$

Appendix E: Proof of Proposition 1

 $\begin{aligned} &(i)\frac{\partial w_{M}^{H}}{\partial \beta_{H}} > 0, \frac{\partial w_{C}^{L}}{\partial \beta_{H}} > 0, \text{ so } w_{i}^{H}, i \in \{M, C\} \text{ increases with } \beta_{H}. \end{aligned}$ $(ii) \text{When } \beta_{L} < \beta_{H} < \frac{3}{2}\beta_{L}, \frac{\partial w_{M}^{L}}{\partial \beta_{H}} < 0, \frac{\partial w_{C}^{L}}{\partial \beta_{H}} < 0, \text{ so } w_{i}^{L}, i \in \{M, C\} \text{ decreases with } \beta_{H}. \text{ When } \frac{3}{2}\beta_{L} < \beta_{H} < 2\beta_{L}, \text{ when } b \text{ is very low, } \frac{\partial w_{M}^{L}}{\partial \beta_{H}} > 0, \frac{\partial w_{C}^{L}}{\partial \beta_{H}} > 0, \text{ so } w_{i}^{L}, i \in \{M, C\} \text{ increases with } \beta_{H}; \text{ when } b \text{ is very high, } \frac{\partial w_{M}^{L}}{\partial \beta_{H}} < 0, \frac{\partial w_{C}^{L}}{\partial \beta_{H}} < 0, \text{ so } w_{i}^{L}, i \in \{M, C\} \text{ decreases with } \beta_{H}. \text{ When } 2\beta_{L} < \beta_{H} < 3\beta_{L}, \frac{\partial w_{M}^{L}}{\partial \beta_{H}} > 0, \frac{\partial w_{C}^{L}}{\partial \beta_{H}} > 0, \text{ so } w_{i}^{L}, i \in \{M, C\} \text{ decreases with } \beta_{H}. \end{aligned}$ $(iii) \frac{\partial w_{H}^{H}}{\partial \beta_{L}} < 0, \frac{\partial w_{C}^{H}}{\partial \beta_{L}} < 0, \text{ so } w_{i}^{H}, i \in \{M, C\} \text{ decreases with } \beta_{L}.$ $(iv) \frac{\partial w_{M}^{L}}{\partial \beta_{K}} > 0, \frac{\partial w_{C}^{L}}{\partial \beta_{K}} > 0, \text{ so } w_{i}^{L}, i \in \{M, C\} \text{ increases with } \beta_{L}.$

Appendix F: Proof of Proposition 2

Define $\Delta \Pi_{MB} = \Pi_M - \Pi_B$. By differentiating *c*, we have $\frac{\partial \Delta \Pi_{MB}}{\partial c} < 0$. Thus, we can get an unique c_{MB} by solving $\Pi_M = \Pi_B$. When $c > c_{MB}$, $\Pi_M < \Pi_B$; When $c < c_{MB}$, $\Pi_M > \Pi_B$. Then we compare the value of c_{MB} with 0 and 1. When $c_{MB} > 1$, $\Pi_M > \Pi_B$, in this case $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{MB1}$. When $c_{MB} < 0$, $\Pi_M < \Pi_B$, in this case $s_{MB2} < s < s_{MB1}$. $\frac{(v+v_0)^2}{16(v-\gamma)}$. When $0 < c_{MB} < 1$, $\Pi_M > \Pi_B$ when $c \in (0, c_{MB})$; $\Pi_M < \Pi_B$ when $c \in (c_{MB}, 1)$, in this case $s_{MB1} < s < 1$ S_{MB2} .

Appendix G: Proof of Proposition 3

Define $\Delta \Pi_{CB} = \Pi_C - \Pi_B$. By differentiating *c*, we have $\frac{\partial \Delta \Pi_{CB}}{\partial c} < 0$. Thus, we can get an unique c_{CB} by solving $\Pi_C = \Pi_B$. When $c > c_{CB}$, $\Pi_C < \Pi_B$; When $c < c_{CB}$, $\Pi_C > \Pi_B$. Then we compare the value of c_{CB} with 0 and 1. When $c_{CB} > 1$, $\Pi_C > \Pi_B$, in this case $s < -\frac{(v+v_0)^2}{8(v-\gamma)}$, the social platform cannot gain profit under the brokerage model, so the comparison makes no sense, and we don't consider this condition. When $c_{CB} < 0$, $\Pi_C < \Pi_B$, in this case $s_{CB} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$. When $0 < c_{CB} < 1$, $\Pi_C > \Pi_B$ when $c \in (0, c_{CB})$; $\Pi_C < \Pi_B$ when $c \in (c_{CB}, 1)$, in this case $-\frac{(v+v_0)^2}{8(v-v)} < s < s_{CB}.$

Appendix H: Proof of Proposition 4

$$\begin{array}{l} (i) \begin{array}{l} \frac{\partial \Delta \Pi_{MB}}{\partial \beta_{H}} = \frac{2b(v+v_{0})^{2}(4v-4\gamma+3c\bar{\beta})[3c(1-b)\beta_{L}(\beta_{H}-\beta_{L})+4(v-\gamma)\beta_{H}]}{[8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}} > 0, \text{ so } \Delta \Pi_{MB} \text{ increases with } \beta_{H}. \end{array} \\ (ii) \begin{array}{l} \frac{\partial \Delta \Pi_{MB}}{\partial \beta_{L}} = -\frac{2(1-b)(v+v_{0})^{2}(4v-4\gamma+3c\bar{\beta})[3cb\beta_{H}(\beta_{H}-\beta_{L})-4(v-\gamma)\beta_{L}]}{[8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}} \\ \gamma, \frac{\partial \Delta \Pi_{MB}}{\partial \beta_{L}} > 0, \Delta \Pi_{MB} \text{ increases with } \beta_{L}. \text{ When } v < \frac{3cb\beta_{H}(\beta_{H}-\beta_{L})}{4\beta_{L}} + \gamma, \frac{\partial \Delta \Pi_{MB}}{\partial \beta_{L}} < 0, \Delta \Pi_{MB} \text{ decreases with } \beta_{L}. \end{array} \\ (iii) \begin{array}{l} \frac{\partial \Delta \Pi_{MB}}{\partial b} = -\frac{(v+v_{0})^{2}(\beta_{H}-\beta_{L})(4v-4\gamma+3c\bar{\beta})[3c(b\beta_{H}+b\beta_{L}-\beta_{L})(\beta_{H}-\beta_{L})-4(v-\gamma)(\beta_{H}+\beta_{L})]}{[8(2v-2\gamma+c\beta_{L})(v-\gamma+c\beta_{L})-bc(\beta_{H}-\beta_{L})(c\beta_{H}-17c\beta_{L}-24v+24\gamma)+9b^{2}c^{2}(\beta_{H}-\beta_{L})^{2}]^{2}} \\ \frac{3c[b\beta_{H}-(1-b)\beta_{L}](\beta_{H}-\beta_{L})}{4(\beta_{H}+\beta_{L})} + \gamma, \frac{\partial \Delta \Pi_{MB}}{\partial b} > 0, \Delta \Pi_{MB} \text{ increases with } b. \end{array} \\ When v < \frac{3c(b\beta_{H}-(1-b)\beta_{L})(\beta_{H}-\beta_{L})}{4(\beta_{H}+\beta_{L})} + \gamma, \frac{\partial \Delta \Pi_{MB}}{\partial b} > 0, \Delta \Pi_{MB} \text{ increases with } b. \end{array} \\ When v < \frac{3c(b\beta_{H}-(1-b)\beta_{L})(\beta_{H}-\beta_{L})}{4(\beta_{H}+\beta_{L})} + \gamma, \frac{\partial \Delta \Pi_{MB}}{\partial b} > 0 \end{array}$$

Appendix I: Proof of Proposition 5

$(i)^{\dot{c}}$	$\Delta \Pi_{CB} =$	$= \frac{2b(1-c)^2\tau^2(v+v_0)^2[4v-4\gamma+3c\tau(1-c)\beta][3c\tau(1-c)(1-b)\beta_L(\beta_H-\beta_L)+4(v-\gamma)\beta_H]}{2b(1-c)^2\tau^2(v+v_0)^2[4v-4\gamma+3c\tau(1-c)\beta][3c\tau(1-c)(1-b)\beta_L(\beta_H-\beta_L)+4(v-\gamma)\beta_H]} > 0 \text{ so } A\Pi_{ab} \text{ increases with } \beta_{ab}$
(1)	$\partial \beta_H$ –	W^2 $V_{0,30} H_{CB}$ increases with p_H .
(ii)	$\partial \Delta \Pi_{CB}$	$= -\frac{2(1-b)(1-c)^{2}\tau^{2}(v+v_{0})^{2}[4v-4\gamma+3c\tau(1-c)\overline{\beta}][3cb\tau(1-c)\beta_{H}(\beta_{H}-\beta_{L})-4(v-\gamma)\beta_{L}]}{when \ \nu > \frac{3cb\tau(1-c)\beta_{H}(\beta_{H}-\beta_{L})}{2} + \frac{3cb\tau(1-c)\beta_{H}(\beta_{H}-\beta$
(11)	$\partial \beta_L$	W^2 , when V^2 $4\beta_L$
	$\gamma, \frac{\partial \Delta \Pi}{\partial \beta}$	$\frac{\delta CB}{\delta_L} > 0$, $\Delta \Pi_{CB}$ increases with β_L . When $v < \frac{3cb\tau(1-c)\beta_H(\beta_H-\beta_L)}{4\beta_L} + \gamma$, $\frac{\partial\Delta \Pi_{CB}}{\partial\beta_L} < 0$, $\Delta \Pi_{CB}$ decreases with β_L .
(iii)	$\partial \Delta \Pi_{C}$	$E_{B} = -\frac{(1-c)^{2}\tau^{2}(v+v_{0})^{2}(\beta_{H}-\beta_{L})[4v-4\gamma+3c\tau(1-c)\overline{\beta}][3\tau(1-c)c(b\beta_{H}+b\beta_{L}-\beta_{L})(\beta_{H}-\beta_{L})-4(v-\gamma)(\beta_{H}+\beta_{L})]}{\text{when}} \text{when} v > 1$
) db	$ W^2$, when V
	$3\tau(1-c)$	$\frac{\partial c(b\beta_H + b\beta_L - \beta_L)(\beta_H - \beta_L)}{\partial t} + \chi \frac{\partial \Delta \Pi_{CB}}{\partial t} > 0$ $\Lambda \Pi_{ce}$ increases with b When $\mu < \frac{3\tau(1-c)c(b\beta_H + b\beta_L - \beta_L)(\beta_H - \beta_L)}{\partial t} + \chi$
		$4(\beta_H+\beta_L)$ + $r, \partial b$ + σ, Mr_{CB} increases with b , when $b < 4(\beta_H+\beta_L)$ + r
	<u>∂∆П_{СВ}</u> ∂b	< 0 , $\Delta \Pi_{CB}$ decreases with b ,
	where	$W = 8[2v - 2\gamma + c(1 - c)\tau\beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}] - bc(1 - c)\tau(\beta_{H} - \beta_{L})[c(1 - c)\tau\beta_{H} - \beta_{L}][v - \gamma + c(1 - c)\tau\beta_{L}]]$
17	c(1 - c)	$c(r)\tau\beta_L - 24\nu + 24\gamma] + 9b^2c^2\tau^2(1-c)^2(\beta_H - \beta_L)^2.$

Appendix J: Proof of Proposition 6

Customer surplus under model $i (i \in \{M, B\})$ can be formulated as $CS_{M} = \int_{\theta_{M}^{*}}^{1} (v_{0} + \theta_{M}v + \gamma D_{M} - cQ_{M} - p_{M}) d\theta_{M} = \frac{2v(v + v_{0})^{2}(4v - 4\gamma + 3c\overline{\beta})^{2}}{[8(2v - 2\gamma + c\beta_{L})(v - \gamma + c\beta_{L}) - bc(\beta_{H} - \beta_{L})(c\beta_{H} - 17c\beta_{L} - 24v + 24\gamma) + 9b^{2}c^{2}(\beta_{H} - \beta_{L})^{2}]^{2}},$ $CS_{B} = \int_{\theta_{B}^{*}}^{1} (v_{0} + \theta_{B}v + \gamma D_{B} - p_{B}) d\theta_{B} = \frac{v(v + v_{0})^{2}}{32(v - \gamma)^{2}}.$ Define $\Delta CS_1 = CS_M - CS_B$. By differentiating c, we have $\frac{\partial \Delta CS_1}{\partial c} < 0$. Thus, when c = 0, ΔCS_1 have the maximum value, and $\Delta CS_1 = \frac{3v(v+v_0)^2}{32(v-\gamma)^2} > 0$. when c = 1, ΔCS_1 have the minimum value. And we have when v > 0 $\frac{\sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{4} + \gamma, \ \Delta CS_1 > 0, \ CS_M > CS_B. \ \text{When} \ \frac{\sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{4} + \gamma > \nu > \gamma, \ \Delta CS_1 < 0, \ \text{so there exists} \ c_0 \in (0, 1) \ \text{by solving} \ \Delta CS_1 = 0. \ \Delta CS_1 > 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ \Delta CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ CS_1 < 0 \ (\text{i.e., } CS_M > CS_B) \ \text{when} \ c \in (0, c_0), \ CS_1 < 0 \ (\text{i.e., } CS$ $\Delta CS_1 < 0$ (i.e., $CS_M < CS_B$) when $c \in (c_0, 1)$.

Appendix K: Proof of Proposition 7

Customer surplus under the CPC advertising model can be formulated as $CS_C = \int_{\theta_c^*}^1 (v_0 + \theta_C v + \gamma D_C - cQ_C - p_C) d\theta_C =$

 $\frac{2v(v+v_0)^2[3c(1-c)\tau\bar{\beta}+4v-4\gamma]^2}{\{8[2v-2\gamma+c(1-c)\tau\beta_L][v-\gamma+c(1-c)\tau\beta_L]-bc(1-c)\tau(\beta_H-\beta_L)[c(1-c)\tau\beta_H-17c(1-c)\tau\beta_L-24v+24\gamma]+9b^2c^2\tau^2(1-c)^2(\beta_H-\beta_L)^2\}^2}$ Define $\Delta CS_2 = CS_C - CS_B$. By differentiating c, we have $\frac{\partial\Delta CS_2}{\partial c} < 0$ when $c < \frac{1}{2}$, and $\frac{\partial\Delta CS_2}{\partial c} > 0$ when $c > \frac{1}{2}$. Thus, when c = 0 or c = 1, ΔCS_2 have the maximum value, and $\Delta CS_2 = \frac{3\nu(\nu+\nu_0)^2}{32(\nu-\nu)^2} > 0$. when $c = \frac{1}{2}$, ΔCS_2 have the minimum value. And we have when $v > \frac{\tau \sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{16} + \gamma, \Delta CS_2 > 0, CS_C > CS_B$. When $\gamma < \nu < \frac{\tau \sqrt{8\beta_L^2 - b(\beta_H - \beta_L)(\beta_H - 17\beta_L) + 9b^2(\beta_H - \beta_L)^2}}{16} + \gamma, \Delta CS_2 < 0, \text{ so there exist } c_1 \in (0, \frac{1}{2}) \text{ and } c_2 \in (\frac{1}{2}, 1) \text{ by solving } \Delta CS_1 = 0. \ \Delta CS_2 > 0 \text{ (i.e., } CS_C > CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (0, c_1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (c_2, 1) \text{ or } c \in (c_2, 1), \ \Delta CS_2 < 0 \text{ (i.e., } CS_C < CS_B) \text{ when } c \in (c_2, 1) \text{ or } c \in (c_2, 1)$ $(c_1, c_2).$

Appendix L: Proof of Proposition 8

The profit allocation in the supply chain under model i ($i \in \{M, C, B\}$) can be formulated as $\lambda_M = \frac{\pi_M^*}{\pi_M^* + \pi_M^*} = \frac{1}{1 + \frac{4(c\beta + \nu - \gamma)(3c\beta + 4\nu - 4\gamma)^2}{(b\beta_H^2 - b\beta_L^2 + \beta_L^2)[8(2\nu - 2\gamma + c\beta_L)(\nu - \gamma + c\beta_L) - bc(\beta_H - \beta_L)(c\beta_H - 17c\beta_L - 24\nu + 24\gamma) + 9b^2c^2(\beta_H - \beta_L)^2]}},$ $\lambda_C = \frac{\pi_C^*}{\pi_C^* + \pi_C^*} =$

$$\begin{split} & \lambda_{C} - \frac{1}{\pi_{C}^{*} + \pi_{C}^{*}} - \frac{1}{\frac{4[c(1-c)\tau\bar{\beta}+v-\gamma][3c(1-c)\tau\bar{\beta}+4v-4\gamma]^{2}}{1 + \frac{4[c(1-c)\tau\bar{\beta}+v-\gamma][3c(1-c)\tau\bar{\beta}+4v-4\gamma]^{2}}{1 + \frac{4[c(1-c)\tau\bar{\beta}+v-\gamma][3c(1-c)\tau\bar{\beta}+4v-4\gamma]^{2}}{1 + \frac{1}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})(8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H} - \beta_{L})[c(1-c)\tau\beta_{H} - 17c(1-c)\tau\beta_{L} - 24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H} - \beta_{L})^{2}}}{1 + \frac{1}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})(8[2v-2\gamma+c(1-c)\tau\beta_{L}][v-\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H} - \beta_{L})[c(1-c)\tau\beta_{H} - 17c(1-c)\tau\beta_{L} - 24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H} - \beta_{L})^{2}}}{1 + \frac{1}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})(8[2v-2\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H} - \beta_{L})[c(1-c)\tau\beta_{H} - 17c(1-c)\tau\beta_{L} - 24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H} - \beta_{L})^{2}}}{1 + \frac{1}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} + \beta_{L}^{2})(8[2v-2\gamma+c(1-c)\tau\beta_{L}] - bc(1-c)\tau(\beta_{H} - \beta_{L})[c(1-c)\tau\beta_{H} - 17c(1-c)\tau\beta_{L} - 24v+24\gamma] + 9b^{2}c^{2}\tau^{2}(1-c)^{2}(\beta_{H} - \beta_{L})^{2}}}{1 + \frac{1}{\tau^{2}(1-c)^{2}(b\beta_{H}^{2} - b\beta_{L}^{2} - b\beta_{L}^$$

- (1)Define $\Delta_1 = \lambda_M \lambda_B$. By differentiating c, we have $\frac{\partial \Delta_1}{\partial c} < 0$. Thus, we can get an unique c_M by solving $\lambda_M = \lambda_B$. When $c > c_M$, $\lambda_M < \lambda_B$; When $c < c_M$, $\lambda_M > \lambda_B$. Then we compare the value of c_M with 0 and 1. When $c_M > 1$, $\lambda_M > \lambda_B$, in this case $-\frac{(v+v_0)^2}{8(v-\gamma)} < s < s_{M1}$. When $c_M < 0$, $\lambda_M < \lambda_B$, in this case $s_{M2} < s < \frac{(v+v_0)^2}{16(v-\gamma)}$. When $0 < s_M < \lambda_B$. $c_M < 1$, $\lambda_M > \lambda_B$ when $c \in (0, c_M)$; $\lambda_M < \lambda_B$ when $c \in (c_M, 1)$, in this case $s_{M1} < s < s_{M2}$.
- (2)Define $\Delta_2 = \lambda_C \lambda_B$. By differentiating c, we have $\frac{\partial \Delta_2}{\partial c} < 0$. Thus, we can get an unique c_C by solving $\lambda_C = \lambda_B$. When $c > c_C$, $\Pi_C < \Pi_B$; When $c < c_C$, $\Pi_C > \Pi_B$. Then we compare the value of c_C with 0 and 1. When $c_C > 1$, $\Pi_C > \Pi_B$, in this case $s < -\frac{(v+v_0)^2}{8(v-\gamma)}$, the social platform cannot gain profit under the brokerage model, so the comparison makes no sense, and we don't consider this condition. When $c_c < 0$, $\Pi_c < \Pi_B$, in this case $s_c < 0$ $s < \frac{(v+v_0)^2}{16(v-\gamma)}$. When $0 < c_c < 1$, $\Pi_c > \Pi_B$ when $c \in (0, c_c)$; $\Pi_c < \Pi_B$ when $c \in (c_c, 1)$, in this case $-\frac{(v+v_0)^2}{8(v-\gamma)} < 1$ $s < s_c$.