HOW SHOULD BRAND MANUFACTURER COOPERATE WITH INFLUENCER IN LIVE STREAMING E-COMMERCE?[1]

Li Cui
Business School
Beijing Technology and Business University
NO.11, Fucheng Road, Haidian District, Beijing 100048, China
cuili629@163.com

Xinzhu Li
Business School
Beijing Technology and Business University
NO.11, Fucheng Road, Haidian District, Beijing 100048, China
lxz1434819946@163.com

Ke Yan[2]
Business School
Beijing Technology and Business University
NO.11, Fucheng Road, Haidian District, Beijing 100048, China
keyan@btbu.edu.cn

T.C.E. Cheng
Department of Logistics and Maritime Studies
The Hong Kong Polytechnic University
11 Yucai Road, Hung Hom, Hong Kong
edwin.cheng@polyu.edu.hk

ABSTRACT

With the flourishing of live streaming e-commerce, many brand manufacturers have sought to increase product sales and profits through live streaming. However, a high return rate, mainly caused by substandard product quality, is a thorny issue for the live e-commerce industry. In this study we develop game-theoretic models to explore the optimal cooperation strategies for a brand manufacturer and an influencer that improve profits and reduce the return rate by considering heterogeneous consumers' purchase and return behaviour. We also conduct a sensitivity analysis on the proportion of followers and consumers' sensitivity to product quality. In addition, we extend the models to explore other issues. We find that the cooperation between the influencer and brand manufacturer to improve product quality will lead to a lower return rate, and higher product quality, sales, and profits. Specifically, when the commission rate is relatively high and the percentage of the influencer's quality investment is relatively low, both the brand manufacturer and influencer will gain more profits. When the influencer endogenously determines the quality of the product, quality cooperation increases the influencer's profit but decreases the brand manufacturer's profit. Long-term cooperation increases the influencer's profit, while the impact on the brand manufacturer's profit is influenced by the live streaming spillover effect.

Keywords: Live streaming e-commerce; Heterogeneous consumers; Product returns; Quality cooperation; Game theory

1. Introduction

1.1. Background and motivation

With the popularity of e-commerce and changes in consumption pattern, live streaming e-commerce has driven

[1] **Cite**: Cui, L., Li, X., Yan, K., & Cheng, T. C. E., (2024, May) How Should Brand Manufacturer Cooperate with Influencer in Live Streaming E-commerce? *Journal of Electronic Commerce Research*, *25*(2).
[2] Corresponding author

a boom in the e-commerce economy as a new marketing mode (Zhang et al., 2020). According to iResearch, the market size of live streaming e-commerce in China reached 1.2 trillion yuan in 2020, and was estimated to rise to 3.5 trillion yuan in 2022[3]. Radically, live streaming is an original channel to create business value in which live streamers can vividly display products and respond to consumers' questions in real time (Fan et al., 2022). In particular, the price advantage and the atmosphere of competition during live streaming effectively stimulate consumers' desire to purchase (Liu et al., 2022).

Seeing that the live streaming e-commerce industry possesses great market potential, many brand manufacturers are willing to cooperate with influencers that have the potential to endorse products. Typically, Internet celebrities or stars whose followers have built up trust in them and will be easily influenced by their attitude towards the brands they introduce have become first-choice influencers (De Veirman et al., 2017; Chia et al., 2021; Dhun & Dangi, 2023). For instance, Austin Li, an Internet celebrity with more than 60 million followers achieved 71-million-yuan sales through live streaming e-commerce, which accounts for nearly half of the annual sales of Winona freeze-dried masks. Nailiang Jia, a star with a large number of followers, achieved 6-million-yuan sales of Ribecs' mask in the live streaming room. When purchasing products in the live streaming room, followers may generate emotional identification with the influencer due to the attractiveness, novelty, and specialization of influencer-generated content, producing a higher value identification of the products that motivates them to buy the products (Farivar et al., 2022; Ye et al., 2021; Aksoy et al., 2023). Nevertheless, due to heterogeneous consumer purchasing behaviour, influencers cannot affect and generate emotional identification in all the consumers. Given the product characteristics, the non-followers that dislike the live streamers' marketing way but like the products will evaluate the products based on price and quality and decide whether to purchase (Hua et al., 2021).

Although live streaming e-commerce motivates consumers to buy products through price advantage and attractive marketing, it simultaneously gives rise to high return rates due to substandard product quality. The average return rate of traditional e-commerce is about 10 to 15 per cent, while the return rate of live streaming e-commerce is generally around 30 to 50 per cent[4]. After experiencing the product, the consumer will probably return it to the brand manufacturer if the product's actual quality does not reach their expected level, which is shaped by the influencer's introduction, online reviews, and previous shopping experience (Taleizadeh et al., 2021). Affected by the issue that Simba has sold fake cubicles through live streaming, many consumers made complaints about substandard product quality. Some influencers have realized the importance of product quality and put more effort into product selection, even participating in the product design, production, and testing processes to provide consumers with high quality-price-ratio products. For example, Austin Li has been deeply involved in the research and development of Florasis' products and effectively monitored product quality, enabling Florasis to provide better cosmetic products and making Florasis rank among the top ten Chinese national cosmetics brands.

The existing research on live streaming e-commerce mainly focuses on consumers' purchase intention (Addo et al., 2021; Sun et al., 2020; Zheng et al., 2022), revenue management (Fan et al., 2022; Mao et al., 2022), and channel selection (Zhang et al., 2024). In addition to considering product price and the live streamer's effort, we consider quality cooperation between the brand manufacturer and influencer. Moreover, we examine the cooperative strategies considering heterogeneous consumers and product returns, and provide insightful guidelines for the decision-maker. Our main research questions are as follows: 1) Under what conditions are the brand manufacturer and influencer profitable? 2) How does cooperation affect the marketing effort, return rate, and profits in the presence of heterogeneous customers? 3) How do the proportion of followers and quality investment affect the brand manufacturer and influencer's optimal decisions?

To address the above questions, we consider cooperation between a brand manufacturer that produces products and an influencer that promotes the products through live streaming. Due to uncertainty of product valuation, the consumers (followers and non-followers) will decide whether to return the product based on their actual utility after receiving and experiencing the product. Given the above considerations, we first develop a game-theoretic model in which the brand manufacturer makes quality investment alone. Then, we develop a quality cooperation model in which the influencer cooperates with the brand manufacturer to improve product quality and decrease the return rate.

1.2. Main contributions and findings

Our main findings are as follows: 1) The brand manufacturer and influencer can cooperate successfully when the return hassle cost, commission rate, and percentage of the influencer's quality investment are relatively small with a reasonable blanket fee. 2) The brand manufacture can improve the product quality and sales, and reduce the return rate by cooperation. However, when the commission rate is relatively high and the percentage of the influencer's quality investment is relatively low, the influencer would like to cooperate. 3) When consumers'

[3] https://report.iresearch.cn/report/202109/3841.shtml
[4] https://wenku.so.com/d/404a5e2391f489abc2c12c6826cb35ef

sensitivity to product quality increases, the brand manufacturer tends to improve product quality. In addition, the brand manufacturer and influencer will gain more profits when the effects of quality investment on the demand and commission rate are relatively large. 4) Increasing the proportion of followers may prompt the influencer to improve the marketing effort to increase product sales. Moreover, the brand manufacturer and influencer will earn more profits when the potential market demand and commission rate are relatively large. 5) When the price is adjustable, the brand manufacturer can upgrade the product, and then raise its price and quality to gain more profit.

We make four contributions in this study as follows: 1) We consider the impacts of heterogeneous consumer return behaviours and product quality on the brand manufacturer and influencer, deriving findings that guide managers to operate more efficiently, advancing existing research. 2) The brand manufacturer and influencer can decrease the product return rate by cooperation. However, whether the influencer can get more profit by cooperation mainly depends on the commission rate and percentage of the influencer's quality investment. 3) We obtain the counter-intuitive finding that the profit of the influencer does not necessarily increase with the proportion of followers. 4) The brand manufacturer should improve the efficiency of quality investment when improving product quality.

We organize the rest of the paper as follows: In Section 2 we review the literature to identify the research gap and position our paper. In Section 3 we introduce the research problem, discuss the assumptions, and formulate the models. In Section 4 we compare the two models and derive the equilibrium outcomes. In Section 5 we discuss the results of numerical studies to generate insights from the analytical findings. In Section 6 we extend the base model to consider three cases where price is a decision variable, expected quality is also a decision variable, and the long-term cooperation is chosen in the second period respectively. Finally, in Section 7, we conclude the paper, discuss the managerial implications of the research findings, and suggest topics for future research.

2. Literature review

In recent years, live streaming e-commerce as a new marketing trend has developed rapidly and attracted much research attention. However, the research on live streaming e-commerce has not been widely studied. Based on existing studies, we aim to explore the optimal decisions considering heterogeneous consumers and return behaviour in live streaming e-commerce. Three streams of literature are closely related to our study, namely quality management, optimal decisions with heterogeneous consumers, and consumer returns in supply chains.

2.1. Quality management

A considerable body of research has considered quality as one of the key factors affecting market demand (Li & Chen, 2020; Liu et al., 2022; Ambilkar et al., 2022). In addition to deriving the quality decisions, we investigate how to improve product quality. Most researchers study quality improvement strategies through contract design. Zhang et al. (2019) found that the fixed fee contract can contribute to higher quality than the revenue sharing contract. Yoo and Cheong (2018) proposed two reward strategies with and without a target quality level to motivate the supplier's quality improvement. Zhou et al. (2022) found that the manufacturer can be motivated to improve product quality through a cost-sharing contract where quality inspection is conducted upon receiving the final product.

There are still no studies on quality improvement in the context of live streaming e-commerce. Similar to Chakraborty et al. (2019), we design a quality cooperation model to improve product quality where the influencer shares a proportion of the brand manufacturer's quality investment. Differently, we consider the relationship between product quality and the return rate, and explore the impact of quality cooperation on the return rate.

2.2. Optimal decisions with heterogeneous consumers

In the literature on supply chain management, there are many studies on the optimal decisions of the supply chain considering heterogeneous consumers. Current studies mostly segment consumers based on their purchase behaviour (Chen et al., 2018; Bagherinejad et al., 2021; Xu et al., 2021; Zhang et al., 2022; Zhu et al., 2023) and product preference (Lv and Li, 2021; Meng et al., 2021a; Meng et al., 2021b; Sarkar & Bhadouriya, 2020). These studies mainly considered the impact of consumer purchase behaviour and product preference on the demand. Several studies typically construct the utility functions of heterogeneous consumers and investigate the impact of the proportion of a particular type of consumers on the decision variables and profit functions. Meng et al. (2021a) divided consumers into green consumers and non-green consumers based on their valuations of green products. They found that the green innovation effort level, environmental benefits, and economic benefits all increase as the proportion of green consumers increases. Sarkar and Bhadouriya (2020) classified consumers into 'traditional' consumers and 'environmentally aware' consumers considering their differences in product preference of green products and non-green products. They concluded that green quality increases but non-green quality decreases with the proportion of 'environmentally aware' consumers.

Different from the above literature, we capture consumers' different reactions to live streaming marketing in the context of live streaming e-commerce, and divide consumers into followers and non-followers. In addition, we extend the research of Hua et al. (2021) and investigate the joint price, quality, and marketing level considering heterogeneous consumers in the context of live streaming e-commerce.

2.3. Consumer returns in supply chains

In the existing literature, most studies focus on the optimal return policy (Alaei et al., 2022; Cao & Choi, 2022; Li et al., 2019; Ren et al., 2021), among which some studies find that the supply chain members can benefit from a full return policy under some circumstances. Combined with the actual situation of live streaming e-commerce, we assume that the brand manufacturer provides a full return policy for consumers. The literature closest to this paper concerns supply chain decisions considering return rate or return volume. Many existing studies have captured the impact of a fixed return rate on supply chain decisions (Fan & Chen, 2020; Li & Liu, 2021; Liu et al., 2023; Radhi & Zhang, 2018; Zhang et al., 2021). Only a few studies construct a function of the return rate or return quantity affected by firms' decisions, i.e., product price (Fan et al., 2022; Hu et al., 2019), refund price (Taleizadeh et al., 2021; Taleizadeh et al., 2020), product quality (Taleizadeh et al., 2021; Taleizadeh et al., 2020), proposing discount (Sadeghi et al., 2019), return quality (Taleizadeh et al., 2021), retailer's effort (Borenich et al., 2020), and live streaming service level (Fan et al., 2022).

Different from the above literature, this study considers the impact of the product price, actual quality, and marketing level on the return rate. Furthermore, based on Wang et al. (2021), our paper constructs different return volume functions of two types of consumers from the perspective of consumer utility to study the impacts of different return behaviours of heterogeneous consumers on the optimal decisions.

In summary, our work differs from the prior literature in two important ways. First, unlike previous studies on live streaming e-commerce, which consider one type of consumers, we classify consumers into followers and non-followers based on whether they are influenced by the influencer's marketing way, and examine the impact of the proportion of followers on the profits and return rate. Second, while previous studies have considered different factors of product returns, we consider the impacts of product price, quality, and influencer's marketing effort on product returns simultaneously from the perspective of consumer utility. Hence, we contribute to the literature by producing findings regarding the interactions between product price, quality, marketing effort, heterogeneous consumers, and product returns. Table 1 summarizes the characteristics of studies that are closely related to our study.

Table 1: Studies that are closely related to our research

Study	Product price	Product quality	Marketing effort	Consumer utility	Heterogeneous consumers	Product returns	Live streaming e-commerce
Mao et al. (2022)	*			*			*
Fan et al. (2022)	*		*	*		*	*
Liu et al. (2022)	*		*			*	*
Zhang et al. (2022)	*		*			*	*
Meng et al. (2021b)	*	*			*		
Hua et al. (2021)	*		*	*	*		
Sarkar and Bhadouriya (2020)	*	*			*		
Taleizadeh et al. (2020)	*	*				*	
Wang et al. (2021)	*			*		*	
Chakraborty et al. (2019)	*	*					
This study	*	*	*	*	*	*	*

3. Models

3.1. Problem description

In this study, we consider that the brand manufacturer cooperates with influencer to sell products to consumers (followers and non-followers) through live streaming. During live streaming, consumers decide whether to buy a product based on product price, quality, and the influencer's marketing effort. Then, consumers will decide whether to keep the product considering the difference between actual quality and expected quality of the product. The decision-making process of consumers is shown in Figure 1.

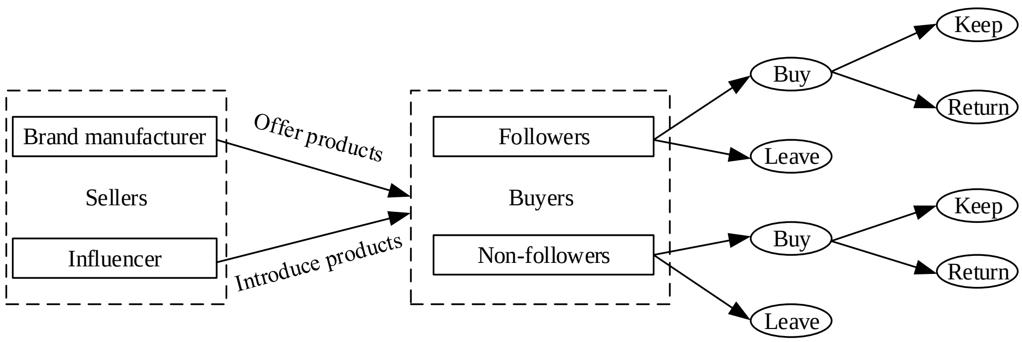


Figure 1: The decision tree of followers and non-followers

There are two scenarios for the cooperation between the brand manufacturer and influencer: without quality cooperation, i.e., ***Scenario 1***, and with quality cooperation, i.e., ***Scenario 2***. Under Scenario 1, the brand manufacturer is responsible for producing, packing, storing, and finally delivering products to consumers that purchase the products shown in the live streaming room, which is similar to Scenario 2. Moreover, the brand manufacturer determines the product quality level and bears the cost of production, inventory, transport, and the extra quality investment. In addition, the influencer focuses on formulating the live streaming scheme, e.g., selecting the live streaming means and creating attractive content. Hence, the influencer contributes the marketing effort and bears the related costs. Specifically, under Scenario 1, the brand manufacturer is solely responsible for product quality, while the influencer is responsible for selling products as a seller. For instance, L'Oreal, a well-established beauty brand with an experienced R&D team, enlists the service of Wangyu Luo, an influencer in the beauty industry, to promote and sell their new products through live streaming. By comparison, the influencer can assist the brand manufacturer in enhancing product quality by sharing a certain proportion of the quality investment under Scenario 2. For example, Brother Yang's live team establishes quality assessment standards and implements multiple testing processes for products sourced from the brand manufacturer. This approach enables the brand manufacturer to enhance quality testing and provide suggestions for quality improvement.

3.2. Assumptions and notations

To develop the game-theoretic model, we introduce the assumptions as follows:

Revenue distribution. If the brand manufacturer and influencer sign a cooperation contract, the brand manufacturer must pay a fixed "blanket fee" F to the influencer before selling products in the live streaming room. The influencer will get a proportional commission ρp for each unit of product sold, where p denotes the product price and ρ denotes the commission rate. In general, the commission rate depending on the category of the product is in the range between 20% and 50%, and the fixed advertising fee relevant to the number of live streamers' fans is at least tens of thousands[5]. In this study we assume that ρ and F are exogenously given. In addition, according to Heydari et al. (2022), we consider the case of a constant product price, which is in line with the actual situation of live e-commerce. For example, famous cosmetics brands such as Florasis and PerfectDiary have made the promise to influencers to keep their product prices unchanged after the live broadcast for a long time.

Consumers. Consumers decide whether to purchase products in the live streaming room through two routes. The first route is product-centred, whereby consumers make purchase decisions based on the features of the products, e.g., quality and price. The second route is social-interaction centred, whereby consumers may be sensitive to the marketing effort and will gain emotional value when purchasing products in the live streaming room (Lu & Chen, 2021; Mohammad et al., 2020). In this study, considering customers' different attitudes towards the influencer's

[5] https://report.iresearch.cn/report/202006/3606.shtml

marketing effort, we classify consumers into two categories: followers and non-followers. We assume that the proportion of followers is θ $(0 < \theta < 1)$, so the proportion of non-followers is $1 - \theta$. The non-followers' purchase decision is influenced by the direct (product-related) factors, i.e., the price and quality of the product, while the followers' purchasing decision is also affected by indirect (non-product-related) factors, i.e., the marketing effort.

Market demand. We apply the consumer choice model (Lin et al., 2020; Fu et al., 2020; Chen & Chen, 2016) to reflect consumers' purchasing behaviour. Let U_l and U_o denote the expected utility of followers and non-followers, respectively. Given that consumers have different preferences for each product, we assume that the consumers' overall valuation of the product ν is uniformly distributed on the line segment between (0,1) (He et al., 2021). According to Ülkü and Gürler (2018), the return hassle cost refers to the expenses of freight, time, anger or disappointment incurred by a consumer who chooses to return a product. When consumers watch a live stream, they often find it challenging to accurately assess the actual value of the product, leaving them uncertain about whether to return the product or keep it. Particularly for products that are inconvenient to move, such as furniture or home appliances, returns will incur higher return hassle costs. In order to avoid the unnecessary return behaviour triggered by impulse consumption, consumers tend to consider the return hassle cost of returning products when purchasing them, thus reducing the possibility of return. Consistent with Ertekin and Agrawal (2021), the return hassle cost can reduce consumers' expected utility. Meanwhile, the marketing effort e can generate emotional value μe among followers, which can increase their expected utility. Furthermore, it is often considered that the price p generates negative utility, and the expected quality Q generates positive utility. Specifically, Q refers to consumers' expectation of product quality based on the live streamer's description of the product and previous consumer experience. Thus, the expected utility of the followers and non-followers are $U_l = \nu - \alpha p + \lambda Q + \mu e - r$ and $U_o = \nu - \alpha p + \lambda Q - r$, respectively, where α is consumers' sensitivity to product price, λ is consumers' sensitivity to product quality, and μ is consumers' sensitivity to marketing. During the live streaming, consumers will purchase the product only if the customer surplus is positive (Webster & Mitra 2007). Therefore, the total market demand of the product is

$$D = \theta A \int_{\alpha p - \mu e - \lambda Q + r}^{1} d\nu + (1-\theta) A \int_{\alpha p - \lambda Q + r}^{1} d\nu = A(1 - \alpha p + \theta \mu e + \lambda Q - r), \quad (1)$$

where A represents the total market size.

Consumer returns. For online shopping, customers cannot touch and feel the real product when deciding whether to buy (Luo & Sun, 2016). Accordingly, the uncertainty of the product cannot be completely eliminated even if the affordance of live streaming e-commerce is more prominent than traditional e-commerce (Hübner et al., 2016; Sun et al., 2020). Generally, there is always a difference between the actual quality q and the expected quality Q of the product. In addition, Mukhopadhyay and Setaputra (2007) found that product quality influences the amount of return directly. When the actual quality level gets closer to or even exceeds the expected quality level, the probability of return will decrease. Moreover, due to the long delivery time, customers' perceived value would reduce when they receive the product, which is a typical psychological phenomenon when making intertemporal choices called preference reversal or inconsistency that can give rise to return behaviour (Hardisty & Pfeffer, 2017; Wang et al., 2021). Following Loewenstein (1988), we denote $\tau\nu$ as the perceived value when customers receive the product, where the time discount factor τ is a constant in the range (0,1). Therefore, the actual utility of the followers and non-followers are $U_l' = \tau\nu - \alpha p + \lambda q + \mu e$ and $U_o' = \tau\nu - \alpha p + \lambda q$ when consumers receive the product. We assume that the returned products would not influence secondary sales (Zhao et al., 2018), which means the brand manufacturer could resell them to other consumers at their original prices. Customers will return the product when the actual utility is negative, and the negative utility of keeping the product is greater than returning the product, i.e., $U_i' < -r$. Therefore, the total amount of return is

$$R = \theta A \int_{\alpha p - \mu e - \lambda Q + r}^{\frac{\alpha p - \mu e - \lambda q - r}{\tau}} d\nu + (1-\theta) A \int_{\alpha p - \lambda Q + r}^{\frac{\alpha p - \lambda q - r}{\tau}} d\nu = A\left(\frac{\alpha p - \theta \mu e - \lambda q - r}{\tau} - \alpha p + \theta \mu e + \lambda Q - r\right). \quad (2)$$

Cost structure. We assume that the brand manufacturer bears the unit production cost c_0 and the unit handling cost of the returned product c_1. The brand manufacturer could repackage, storage and transport, or not reprocess the returned products (Wang et al., 2021). Thus, we assume that the unit production cost of the product is much greater than the unit handling cost of the returned product, i.e., $c_0 \gg c_1$. Also, the brand manufacturer and the influencer will make quality and marketing investments to promote the product. We assume that the actual quality of the product is q $(q > 0)$, which will lead to the cost as σq^2 (Ghosh & Shah, 2015; Heydari et al., 2021). Following Heydari et al. (2021), we let σ be the quality investment coefficient, which is the inverse function of production efficiency for the brand manufacturer. Similarly, we denote the marketing investment as ξe^2, where ξ is the marketing investment coefficient.

Given the above description and assumptions, we summarize in Table 2 the basic parameters and variables of this study.

Table 2: Parameters and variables used in this study

Notation	Description
Parameter	
A	Potential maximum demand
ν	The overall valuation of the product, which is uniformly distributed over an interval $[0,1]$
α	Consumers' sensitivity to product price
μ	Consumers' sensitivity to marketing
λ	Consumers' sensitivity to product quality
τ	Time discount factor, $0 < \tau < 1$
θ	The percentage of followers, $0 < \theta < 1$
r	The return hassle cost
Q	The expected quality of the product
F	Blanket fee
ρ	Commission rate, $0 < \rho < 1$
c_0	The unit production cost of the product
c_1	The unit handling cost of the returned product
σ	The coefficient of quality investment
ξ	The coefficient of marketing investment
p	The product price
ϕ	The percentage of the influencer's quality investment
Decision variable	
q	The actual quality
e	The marketing effort
Derived functions	
Π_M	Profit of the brand manufacturer
Π_L	Profit of the influencer

3.3. Model setup

Based on Scenario 1 and Scenario 2 in Section 3.1. Problem description, we intend to formulate the model without quality cooperation (Model N) and the model with quality cooperation (Model Y). Under Models N and Y, the brand manufacturer M first determines the product quality, then the influencer L determines the marketing effort level. We use backward induction to solve the Stackelberg game between the brand manufacturer (leader) and influencer (follower). We provide the expressions for the threshold values of all the propositions in the Appendix.

Due to the fact that live streaming is periodic, the products purchased and kept by consumers can be counted as sales for a specific live streaming session. However, if the products are returned, the brand manufacturer will inspect their quality and then resell them in the next period. Consequently, returned products are not counted as sales for this period. In China, brand manufacturers offer consumers the no-reason full return policy within seven days to stimulate consumer demand. Once consumers receive the products, they can choose to either "confirm receipt" if they are highly satisfied or wait for the e-commerce platform to automatically "confirm receipt" on their behalf, which signifies a successful transaction. Otherwise, consumers can initiate a return request on the platform and send the product back to the brand manufacturer under the no-reason full return policy. After the brand manufacturer receives the product and verifies the relevant information, the refund will be returned to the consumer, which represents a failing transaction. Therefore, the actual sales is $D - R$. In Model Y, the influencer will bear a proportion of the cost of quality improvement ϕ $(0 < \phi < 1)$, in addition to bearing the full cost of the marketing investment. Thus, the profits of the brand manufacturer and influencer under Model Y are as follows:

$$\Pi_M^Y = ((1-\rho)p - c_0)(D - R) - c_1 R - (1-\phi)\sigma q^2 - F, \tag{3}$$

$$\Pi_L^Y = \rho p(D - R) - \xi e^2 - \phi\sigma q^2 + F. \tag{4}$$

Theorem 1. The optimal solutions under Model Y are

$$q^{Y*} = \frac{A\lambda(p(1-\rho) - c_0 + c_1)}{2\sigma\tau(1-\phi)} \text{ and } e^{Y*} = \frac{A\theta\mu\rho p}{2\xi\tau}.$$

Then, we obtain the equilibrium demands, return volumes, and profits of the brand manufacturer and influencer as follows:

$$D^{Y*} = \frac{A\left(Ap\theta^2\mu^2\rho + 2(1 - r - p\alpha + Q\lambda)\xi\tau\right)}{2\xi\tau}, \tag{5}$$

$$R^{Y*} = \frac{A\left(Ap\left(\lambda^2\xi(1-\rho)+\theta^2\mu^2\rho\sigma(1-\phi)\right)-2\xi\sigma\tau(1-\phi)(r+p\alpha-\tau)-A\lambda^2\xi(c_0-c_1)\right)}{2\xi\sigma\tau^2(1-\phi)}, \tag{6}$$

$$\Pi_M^{Y*} = \frac{A\left(A\lambda^2\xi {I_1}^2+2\left(1-\phi\left(Ap\theta^2\mu^2\rho\sigma I_2-2\xi\sigma\tau(r+p\alpha-\tau)I_1+2\xi\sigma\tau c_1(2r-(1-r-p\alpha+Q\lambda)\tau)\right)\right)\right)}{4\xi\sigma\tau^2(1-\phi)} - F\ , \tag{7}$$

$$\Pi_L^{Y*} = \frac{A\left(A\lambda^2\xi I_1 I_3+\sigma p\rho(1-\phi)^2\left(A\theta^2\mu^2\rho p-4\xi\tau(r+p\alpha-\tau)\right)\right)}{4\xi\sigma\tau^2(1-\phi)^2} + F, \tag{8}$$

where $I_1 = p(1-\rho) - c_0 + c_1$, $I_2 = p(1-\rho) - c_0 + (1-\tau)c_1$ and $I_3 = p(\rho(2-\phi)-\phi) + \phi c_0 - \phi c_1$.

Then, we find the optimal solution of Model N by substituting $\phi = 0$ into the optimal solutions of Model Y.

Theorem 2. The optimal solutions under the Model N are

$$q^{N*} = \frac{A\lambda(p(1-\rho)-c_0+c_1)}{2\sigma\tau} \text{ and } e^{N*} = \frac{A\theta\mu\rho p}{2\xi\tau}.$$

Substituting the optimal solutions into the demand function and return volume function, we obtain the equilibrium demand and return volume as follows:

$$D^{N*} = \frac{A(Ap\theta^2\mu^2\rho+2(1-r-p\alpha+Q\lambda)\xi\tau)}{2\xi\tau}, \tag{9}$$

$$R^{N*} = \frac{A\left(2\xi\sigma\tau\left(p\alpha(1-\tau)+Q\lambda\tau-r(1+\tau)\right)+A\lambda^2\xi(c_0-c_1)-Ap\left(\lambda^2\xi(1-\rho)+\theta^2\mu^2\rho\sigma(1-\tau)\right)\right)}{2\xi\sigma\tau^2}. \tag{10}$$

It follows that the equilibrium profits of the brand manufacturer and influencer are

$$\Pi_M^{N*} = \frac{A\left(A\lambda^2\xi {I_1}^2+2Ap\theta^2\mu^2\rho\sigma I_2-4\xi\sigma\tau(r+p\alpha-\tau)I_1+4\xi\sigma\tau c_1(2r-(1-r-p\alpha+Q\lambda)\tau)\right)}{4\xi\sigma\tau^2} - F, \tag{11}$$

$$\Pi_L^{N*} = \frac{Ap\rho\left(2A\lambda^2\xi I_1+Ap\theta^2\mu^2\rho\sigma-4\xi\sigma\tau(r+p\alpha-\tau)\right)}{4\xi\sigma\tau^2} + F. \tag{12}$$

Next, we conduct a feasibility analysis of the models to yield the following result.

Proposition 1. When the brand manufacturer cooperates with the influencer to improve product quality, they can conduct live streaming successfully when $0 < \rho < \frac{p-c_0+c_1}{p}$ and $0 < r < \frac{Ap\theta^2\mu^2\rho+2(1-p\alpha+Q\lambda)\xi\tau}{2\xi\tau}$. Moreover, the ranges of F and ϕ relevant to ρ are as follows:

(1) When the commission rate is relatively small, i.e., $0 < \rho \le \frac{p-c_0+c_1}{3p}$, we have

① $\hat{F}_1 < F < \hat{F}_3$ and $0 \le \phi < \phi_1$,

② $F > \hat{F}_3$ and $\phi_3 < \phi < \phi_1$.

(2) When the commission rate is relatively large, i.e., $\frac{p-c_0+c_1}{3p} < \rho \le \frac{p-c_0+c_1}{p}$, we have

① $\hat{F}_2 < F < \hat{F}_1$ and $\phi_2 < \phi < \phi_1$,

② $\hat{F}_1 < F < \hat{F}_3$ and $0 \le \phi < \phi_1$,

③ $F > \hat{F}_3$ and $\phi_3 < \phi < \phi_1$.

Whether the brand manufacturer and influencer are profitable mainly depends on the commission rate, blanket fee, return hassle cost, and the percentage of the influencer's quality investment. Specifically, when the commission rate and return hassle cost are relatively low, and the blanket fee and the percentage of the influencer's quality investment are within a reasonable range, both parties are profitable. If the commission rate is relatively large, the brand manufacturer's revenue can hardly cover the high cost, so yielding a negative profit. In addition, if consumers' return hassle cost is higher than the maximum threshold, they will not purchase products. This is why some home appliance brands offer the free door-to-door return service. When the return hassle cost is relatively large, consumers' utility to return the product is lower than their utility to keep it, so the return volume is zero. Therefore, managers should not only adjust the commission rate, blanket fee, and the percentage of the influencer's quality investment reasonably to balance the profits of both parties, but also provide a superior return service as much as possible for consumers to reduce the return hassle cost.

We also find that the blanket fee is related to the commission rate and the percentage of quality investment cost shared by the influencer. When the commission rate is relatively large, if the percentage of the influencer's quality investment is relatively large, the increment of the influencer's revenue driven by product quality improvement is larger than that of the quality investment cost, which gives rise to an increase in the profit of the influencer. So the adjustable range of the blanket fee becomes wider, which means the blanket fee can be minimized to reduce the revenue gap between the brand manufacturer and influencer. Furthermore, when the commission rate is certain, and if the blanket fee is relatively large, the influencer should share more quality investment cost to control product quality, thus achieving the expected product sales. Therefore, the decision-maker should set a reasonable commission rate, blanket fee, and percentage of the influencer's quality investment to strike a balance in profits when both the brand manufacturer and influencer are profitable.

4. Analysis

In this section we first analyze the equilibrium solutions to examine the impacts of consumers' sensitivity to product quality, and the proportion of followers on the optimal decisions of the brand manufacture and influencer, as well as the profits of both parties.

Proposition 2. With increasing consumers' sensitivity to product quality, the quality and actual sales increase, and the marketing effort remains unchanged, while changes in the brand manufacturer's and influencer's profits are affected by the coefficient of quality investment and commission rate.

(1) Model *N*: ① $\frac{\partial e^{N*}}{\partial \lambda} = 0$; ② $\frac{\partial q^{N*}}{\partial \lambda} > 0$; ③ $\frac{\partial (\mathrm{D}^{N*} - \mathrm{R}^{N*})}{\partial \lambda} > 0$; ④ $\frac{\partial \Pi_L^{N*}}{\partial \lambda} > 0$; and ⑤ when $\sigma < \frac{A\lambda(p(1-\rho) - c_0 + c_1)^2}{2Q\tau^2 c_1}$, $\frac{\partial \Pi_M^{N*}}{\partial \lambda}$>0; otherwise, $\frac{\partial \Pi_M^{N*}}{\partial \lambda} < 0$.

(2) Model *Y*: ① $\frac{\partial e^{Y*}}{\partial \lambda} = 0$; ② $\frac{\partial q^{Y*}}{\partial \lambda} > 0$; ③ $\frac{\partial (D^{Y*} - R^{Y*})}{\partial \lambda} > 0$; ④ when $\rho < \frac{\phi(p - c_0 + c_1)}{p(2-\phi)}$, $\frac{\partial \Pi_L^{Y*}}{\partial \lambda} < 0$; otherwise, $\frac{\partial \Pi_L^{Y*}}{\partial \lambda} > 0$; and ⑤ when $\sigma < \frac{A\lambda(p(1-\rho) - c_0 + c_1)^2}{2Q\tau^2(1-\phi)c_1}$, $\frac{\partial \Pi_M^{Y*}}{\partial \lambda} > 0$; otherwise, $\frac{\partial \Pi_M^{Y*}}{\partial \lambda} < 0$.

With increasing consumers' sensitivity to product quality, improvement of product quality decreases the return volume, so the actual sales increase. Meanwhile, when the effect of quality investment is relatively low, if consumers' sensitivity to product quality increases, the increment in the brand manufacturer's revenue will be less than that in cost, which ultimately results in a decrease in the brand manufacturer's profit. This indicates that when consumers become increasingly sensitive to product quality, the brand manufacturer should not only improve product quality, but also improve production efficiency as much as possible to gain more profit.

As for the influencer, when the influencer does not participate in quality cooperation, improvement in product quality increases their revenue while their marketing investment cost remains unchanged, which increases their profit. If the influencer participates in quality cooperation, improvement in product quality raises the quality investment cost, so it is beneficial to the influencer when the commission rate is relatively large. From the above analysis, we see that when consumers' sensitivity to product quality increases, offering the influencer a higher commission rate and reducing the coefficient of quality investment through production optimization is the best way to ensure that improving product quality would improve both parties' profits.

Proposition 3. With increasing proportion of followers, the marketing effort and actual sales increase, and quality keeps unchanged, while changes in the brand manufacturer's and influencer's profits are affected by the commission rate and potential market demand.

(1)$\frac{\partial e^{N*}}{\partial \theta} > 0$, $\frac{\partial e^{Y*}}{\partial \theta} > 0$; (2)$\frac{\partial q^{N*}}{\partial \theta} = 0$, $\frac{\partial q^{Y*}}{\partial \theta} = 0$; (3) $\frac{\partial (D^{N*} - R^{N*})}{\partial \theta} > 0$, $\frac{\partial (D^{Y*} - R^{Y*})}{\partial \theta} > 0$; (4) when A >0.5, $\frac{\partial \Pi_L^{N*}}{\partial \theta} > 0$ and $\frac{\partial \Pi_L^{Y*}}{\partial \theta} > 0$; otherwise, $\frac{\partial \Pi_L^{N*}}{\partial \theta} < 0$ and $\frac{\partial \Pi_L^{Y*}}{\partial \theta} < 0$; and (5) when $\rho < 1 - \frac{c_0 - (1-\tau)c_1}{p}$, $\frac{\partial \Pi_M^{N*}}{\partial \theta} > 0$, and $\frac{\partial \Pi_M^{Y*}}{\partial \theta} > 0$; otherwise, $\frac{\partial \Pi_M^{N*}}{\partial \theta} < 0$ and $\frac{\partial \Pi_M^{Y*}}{\partial \theta} < 0$.

Whether the influencer participates in quality cooperation, both the marketing effort of the influencer and actual product sales increase with the proportion of followers, while product quality remains constant. When the proportion of followers increases, the influencer tends to increase their marketing effort to attract consumers to purchase, which can increase actual sales without additional quality investment. Hence, when the potential market demand is relatively large (small), the influencer's profit increases (decreases). However, when the potential market demand is relatively small, the actual sales are similarly quite low. In the meantime, increasing the proportion of followers causes the increment of revenue to be lower than that of cost, so the influencer's profit will decrease. Moreover, when the commission rate is relatively low, the brand manufacturer's revenue increases and its cost decreases, which will bring an increase in the profit of the brand manufacturer with increasing proportion of followers. Conversely, the brand manufacturer's profit declines. We can conclude that it is not necessarily more profitable for the brand manufacturer to work with the influencer that has a large number of followers. The commission rate for the influencer cannot be too large, and the influencer should consider the match between their image and the product, so that a well-matched influencer means a larger potential demand for the product.

Next, we compare the two models with and without quality cooperation to analyze whether the influencer should participate in quality investment.

Proposition 4. Quality cooperation can improve product quality and actual sales, and decrease return volume and return rate, i.e., ① $q^{N*} < q^{Y*}$; ② $R^{N*} > R^{Y*}$; ③ $D^{N*} - R^{N*} < D^{Y*} - R^{Y*}$; and ④ $\frac{R^{N*}}{D^{N*}} > \frac{R^{Y*}}{D^{Y*}}$.

As the influencer's quality investment activities, i.e., the process of production, design, and testing, can relieve the cost pressure of the brand manufacturer, product quality and actual sales will increase by influencer's participating in quality cooperation. Thus, the brand manufacturer will have extra funds for product quality and provide higher-quality products for consumers. As the cost of quality investment increases, the influencer can maintain the same marketing effort level without investing more in product marketing to achieve a lower return rate and higher actual product sales. The results show that product quality rather than marketing effort is the key factor for brand development. Furthermore, a high order volume during live streaming does not represent high actual product sales. Hence, the brand manufacturer should focus more on the actual sales of products before cooperating with the influencer. However, if the quality of the product is low, the brand manufacturer will fail to improve the actual sales, and may also cause a high return rate that brings reverse logistics pressure to the supply chain.

Proposition 5. Quality cooperation can improve the brand manufacturer's profit. However, the change in the influencer's profit is affected by the commission rate and percentage of the influencer's quality investment as follows:

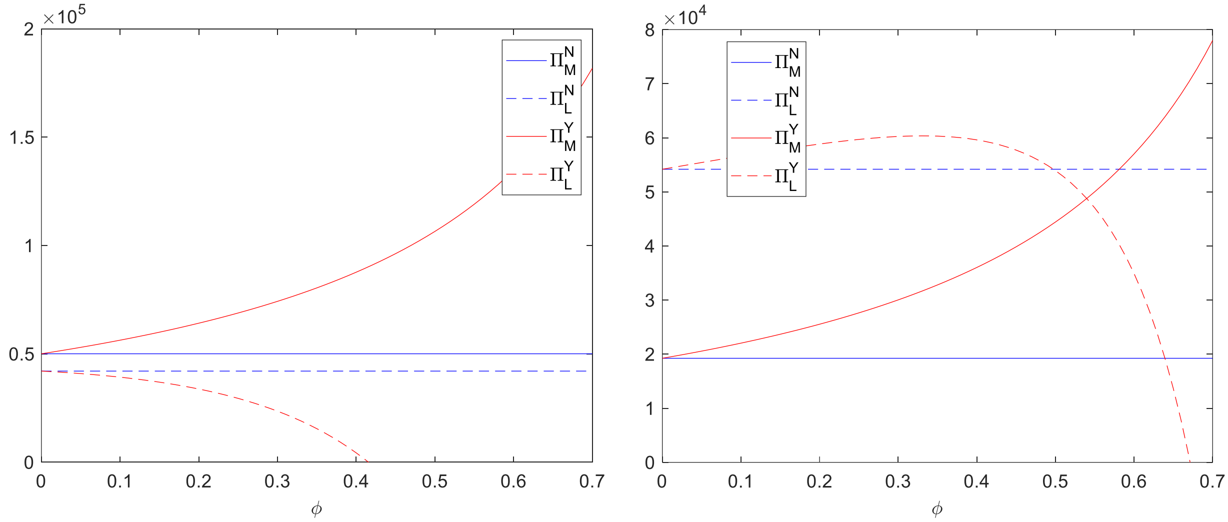
① $\Pi_M^{N*} < \Pi_M^{Y*}$;

② When $0 < \rho < \frac{p-c_0+c_1}{3p}$, $\Pi_L^{N*} > \Pi_L^{Y*}$; when $\frac{p-c_0+c_1}{3p} < \rho < \frac{p-c_0+c_1}{p}$, there are two situations: if $0 < \phi < \frac{p(3\rho-1)+c_0-c_1}{2p\rho}$, $\Pi_L^{N*} < \Pi_L^{Y*}$; if $\frac{p(3\rho-1)+c_0-c_1}{2p\rho} < \phi < 1$, $\Pi_L^{N*} > \Pi_L^{Y*}$.

Under Model *Y*, the influencer's cost-sharing of quality investment can reduce the brand manufacturer's cost, so the brand manufacturer can put more effort in improving product quality to increase actual sales, so obtaining more profit. However, changes in the influencer's profit are influenced by the commission rate and proportion of the influencer's quality investment. Specifically, when the commission rate is relatively low, the increment of the influencer's revenue arising from product quality improvement cannot make up for the loss of their quality investment cost, thus resulting in a decrease in the influencer's profit. When the commission rate is relatively large, if the percentage of the influencer's quality investment is relatively low, the increment of the influencer's revenue is larger than that of the influencer's cost, which will improve the influencer's profit. Conversely, the influencer's profit will decline. Therefore, when the commission rate is relatively large and the percentage of the influencer's quality investment is relatively low, the influencer's involvement in quality cooperation is beneficial to both parties.

5. Numerical studies

Since our models contain several parameters, it is difficult to analyze their impacts on the game outcomes analytically. So, we conduct numerical studies to show related changes. Based on the feasible conditions of the models, we set A=50, α=2, λ=3, μ=1, θ=0.5, Q=10, r=0.01, τ=0.95, c_0=1, c_1=0.01, σ=1, ξ=1, F=5000, p=5, and divide the range of ρ into $0 < \rho < \frac{p-c_0+c_1}{3p}$ and $\frac{p-c_0+c_1}{3p} < \rho < \frac{p-c_0+c_1}{p}$. Setting ρ=0.2 and ρ=0.4, we show the changes in profits in Figures 2 and 3.



ρ=0.2 ρ=0.4

Figure 2: Changes in profits when ρ=0.2 and ρ=0.4

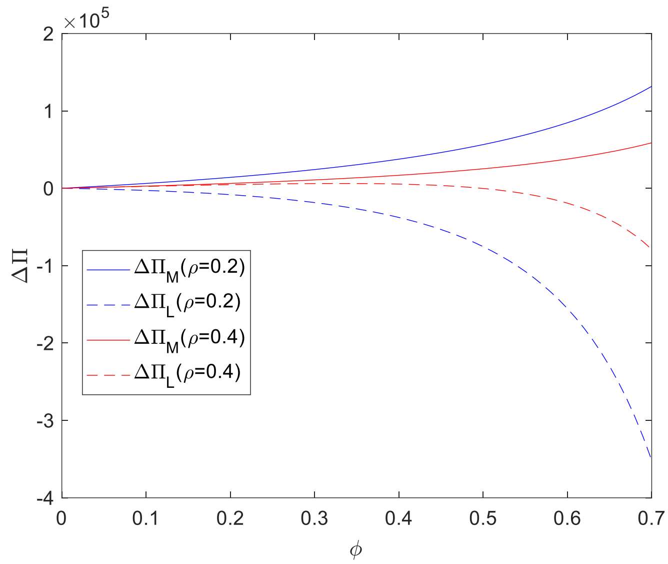


Figure 3: Change in profit after quality cooperation

As shown in Figure 2, when the commission rate equals 0.4, the feasible range of the percentage of the influencer's quality investment is larger. This indicates that when the commission rate is relatively large, the influencer's decision-making about the percentage of the influencer's quality investment will be more flexible. In addition, we also observe that the profit of the brand manufacturer gradually increases with the percentage of the influencer's quality investment within the feasible range, while the changes in the influencer's profit are related to the commission rate.

Comparing the profits of the brand manufacturer and influencer in Figure 2, we conclude that when the commission rate is relatively low, the brand manufacturer's profit is always higher than the influencer's profit regardless of whether the influencer is involved in quality cooperation. In contrast, the brand manufacturer's profit will be lower than the influencer's profit if the influencer does not participate in quality cooperation. And yet, if the influencer participates in quality cooperation, there is a clear dividing point in the comparison of the brand manufacturer's and influencer's profits. Specifically, when the percentage of the influencer's quality investment is less (larger) than the threshold (ϕ<0.54), the profit of the brand manufacturer is lower (higher) than that of the influencer. Form Figure 3, we conclude that the brand manufacturer will definitely gain more profit through quality cooperation. However, the influencer will be more willing to participate in quality cooperation when the commission rate is relatively large and the percentage of the influencer's quality investment is relatively low.

6. Extended model

6.1. Variable pricing

In the above models, we mainly discuss the situation where the brand manufacturer and influencer sign an agreement to guarantee stability of the promotional price, i.e., the price remains unchanged. To extend our research and help managers make better decisions, we consider the situation where the price is variable. The manufacturer first determines the product price and quality, then the influencer determines the marketing effort. The profit functions of the brand manufacturer and influencer are similar to the main model.

When the influencer does not participate in quality cooperation, the optimal decisions of the brand manufacturer and influencer are as follows:

$$p^{N*}=\frac{-2\xi(1-\rho)\sigma\tau^2+A\lambda^2\xi(1-\rho)(c_0-c_1)+(\theta^2\mu^2\rho\sigma-2\alpha\xi\sigma\tau)(c_0-(1-\tau)c_1)}{(1-\rho)(A\lambda^2\xi(1-\rho)+2\theta^2\mu^2\rho\sigma-4\alpha\xi\sigma\tau)},$$

$$q^{N*}=\frac{A\lambda(-2\xi(1-\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1+\tau)c_1))}{2\tau(A\lambda^2\xi(1-\rho)+2\theta^2\mu^2\rho\sigma-4\alpha\xi\sigma\tau)},$$

$$e^{N*}=\frac{A\theta\mu\rho\left(-2\xi(1-\rho)\sigma\tau^2+A\lambda^2\xi(1-\rho)(c_0-c_1)+(\theta^2\mu^2\rho\sigma-2\alpha\xi\sigma\tau)(c_0-(1-\tau)c_1)\right)}{2\xi\tau(1-\rho)(A\lambda^2\xi(1-\rho)+2\theta^2\mu^2\rho\sigma-4\alpha\xi\sigma\tau)}.$$

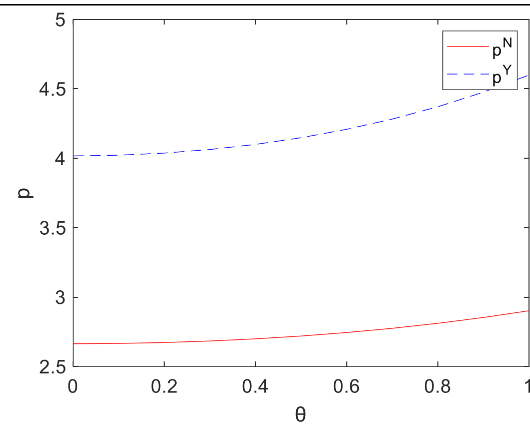
The optimal decisions when the influencer is involved in quality cooperation are as follows:

$$p^{Y*}=\frac{\lambda^2\xi(1-\rho)(c_0-c_1)+\sigma(1-\phi)(-2\xi(1-\rho)\tau^2+(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1-\tau)c_1))}{(1-\rho)(\lambda^2\xi(1-\rho)+2\sigma(\theta^2\mu^2\rho-2\alpha\xi\tau)(1-\phi))},$$

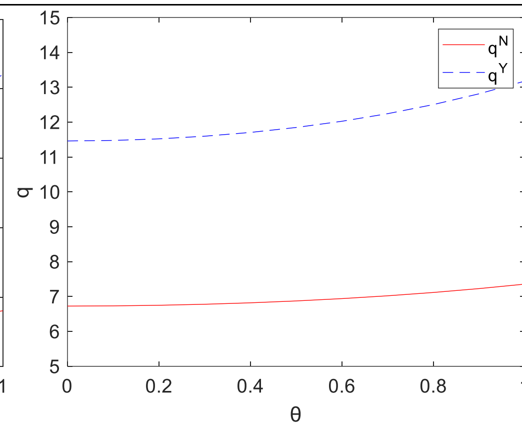
$$q^{Y*}=\frac{A\lambda(-2\xi(1-\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1+\tau)c_1))}{2\tau(\lambda^2\xi(1-\rho)+2\sigma(\theta^2\mu^2\rho-2\alpha\xi\tau)(1-\phi))},$$

$$e^{Y*}=\frac{A\theta\mu\rho\left(\sigma(1-\phi)\left(-2\xi(1-\rho)\tau^2+\left(\theta^2\mu^2\rho-2\alpha\xi\tau\right)(c_0-(1-\tau)c_1)\right)+A\lambda^2\xi(1-\rho)(c_0-c_1)\right)}{2\xi\tau(1-\rho)(\lambda^2\xi(1-\rho)+2\sigma(\theta^2\mu^2\rho-2\alpha\xi\tau)(1-\phi))}.$$

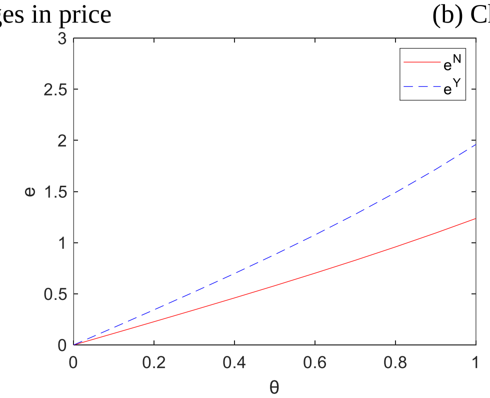
Given the complexity of the analysis, we conduct numerical studies to examine changes in the optimal decisions and profits after the influencer participates in quality cooperation. In the feasible range, we set the base values of the parameters as follows: A=10, α=0.7, λ=0.4, μ=0.4, $\theta\in[0,1]$, Q=20, r=0.01, τ=0.95, c_0=1, c_1=0.01, σ=0.5, ξ=1, ρ=0.3 F=3, and ϕ=0.1. We present the results of the sensitivity and comparative analyses under the two models where we take the proportion of followers θ as an independent variable (see Figures 4-6).



(a) Changes in price

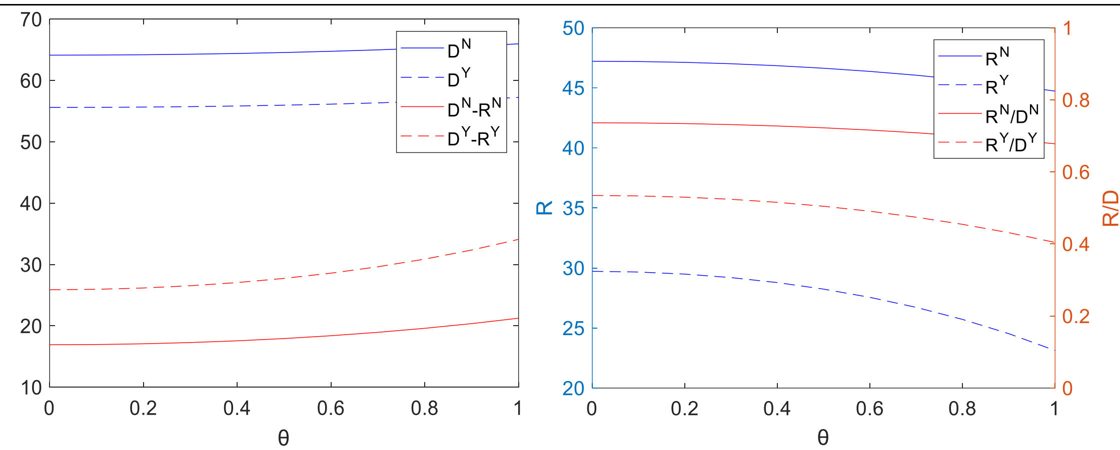


(b) Changes in quality



(c) Changes in marketing effort level

Figure 4: Changes in decision variables with θ



(a) Changes in demand and actual sales (b) Changes in return volume and return rate

Figure 5: Changes in demand, actual sales, return volume and return rate with θ

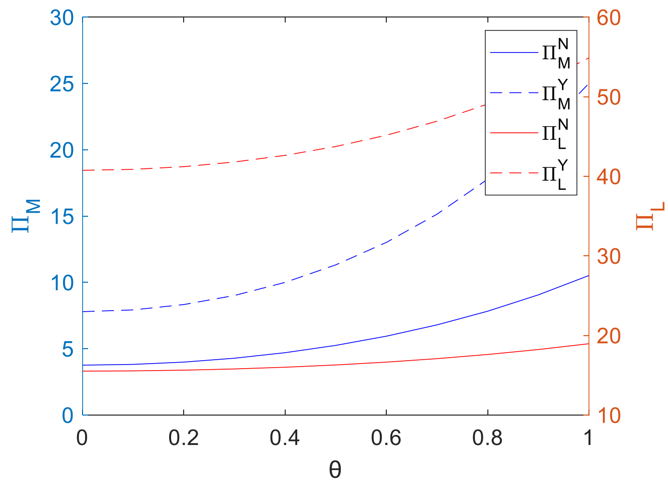


Figure 6: Changes in profits of the brand manufacturer and influencer with θ

From Figures 4 and 5, we conclude that the price, quality, marketing effort and actual sales all increase when the influencer participates in quality cooperation, and the return rate decreases. Since the price increases with quality improvement, the influencer needs to increase the marketing effort to let consumers know the difference in the product before and after quality improvement, which helps mitigate consumers' concern about product quality and dissatisfaction with increasing product price. Furthermore, quality cooperation does not necessarily increase the product demand, but increases actual sales. From Figure 6, we also find that quality cooperation can improve the profits of the brand manufacturer and influencer, and is more beneficial to the influencer.

6.2. Variable expected quality

Due to the uncertainty of product quality, the expected quality of a product normally deviates from its actual quality. In Extended Model 2, we assume that $Q = kq$, where k denotes the deviation of the expected quality from the actual quality. $k > 1$ indicates that the expected quality is higher than the actual quality, while $k < 1$ indicates that the expected quality is lower than the actual quality. Substituting k into Eqs (1) and (2), we find

$$D = A(1 - \alpha p + e\theta\mu + \lambda kq - r), \tag{13}$$

$$R = A(\frac{\alpha p - \theta\mu e - \lambda q}{\tau} - \alpha p + \theta\mu e + \lambda kq - r). \tag{14}$$

Then, substituting Eqs (13) and (14) into the profit functions Eqs (3) and (4), and using backward induction, we derive the optimal solutions under Model Y as follows:

$$p^{Y*} = \frac{A\lambda^2\xi(-1+\rho)(c_0-(1-k\tau)c_1)+\sigma(1-\phi)(2\xi(1-\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1-\tau)c_1))}{(1-\rho)(2\sigma(2\alpha\xi\tau-\theta^2\mu^2\rho)(1-\phi)-A\lambda^2\xi(1-\rho))},$$

$$q^{Y*} = \frac{A\lambda(2\xi(1-\rho)\tau^2+(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0+(-1+(-1+2k)\tau)c_1))}{2\tau(2\sigma(2\alpha\xi\tau-\theta^2\mu^2\rho)(1-\phi)-A\lambda^2\xi(1-\rho))},$$

$$e^{Y*} = \frac{A\theta\mu\rho(A\lambda^2\xi(-1+\rho)(c_0+(-1+k\tau)c_1)+\sigma(-2\xi(-1+\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0+(-1+\tau)c_1)))}{2\xi\tau(1-\rho)(2\sigma(2\alpha\xi\tau-\theta^2\mu^2\rho)(1-\phi)-A\lambda^2\xi(1-\rho))}.$$

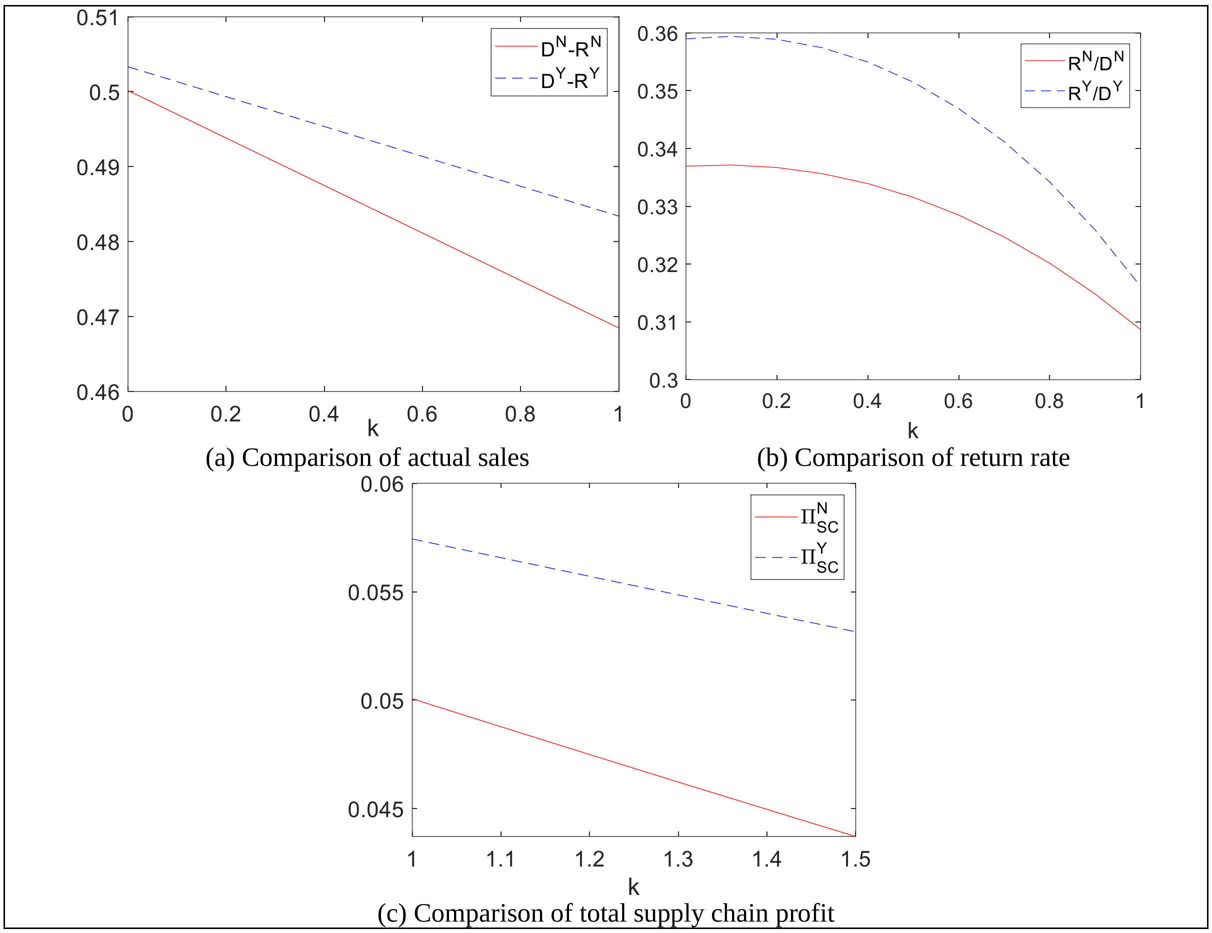
Letting ϕ=0, we find the optimal solutions under Model N as follows:

$$p^{N*} = \frac{A\lambda^2\xi(-1+\rho)(c_0-(1-k\tau)c_1)+\sigma(2\xi(1-\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1-\tau)c_1))}{(1-\rho)(A\lambda^2\xi(-1+\rho)-2\theta^2\mu^2\rho\sigma+4\alpha\xi\sigma\tau)},$$

$$q^{N*} = \frac{A\lambda(2\xi(1-\rho)\tau^2+(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0+(-1+(-1+2k)\tau)c_1))}{2\tau(A\lambda^2\xi(-1+\rho)-2\theta^2\mu^2\rho\sigma+4\alpha\xi\sigma\tau)},$$

$$e^{N*} = \frac{A\theta\mu\rho(A\lambda^2\xi(-1+\rho)(c_0+(-1+k\tau)c_1)+\sigma(-2\xi(-1+\rho)\tau^2-(\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0+(-1+\tau)c_1)))}{2\xi\tau(1-\rho)(A\lambda^2\xi(-1+\rho)-2\theta^2\mu^2\rho\sigma+4\alpha\xi\sigma\tau)}.$$

Next, we conduct numerical studies to compare the actual sales, return rates, and total supply chain profits of the two models when $k < 1$ or $k > 1$. Setting A=8, α=0.7, λ=0.3, μ=0.4, θ=0.5, r=0.01, τ=0.95, c_0=1, c_1=0.01, σ=1, ξ=1, ρ=0.3 F=3, and ϕ=0.3, we obtain the results shown in Figures 7 and 8.

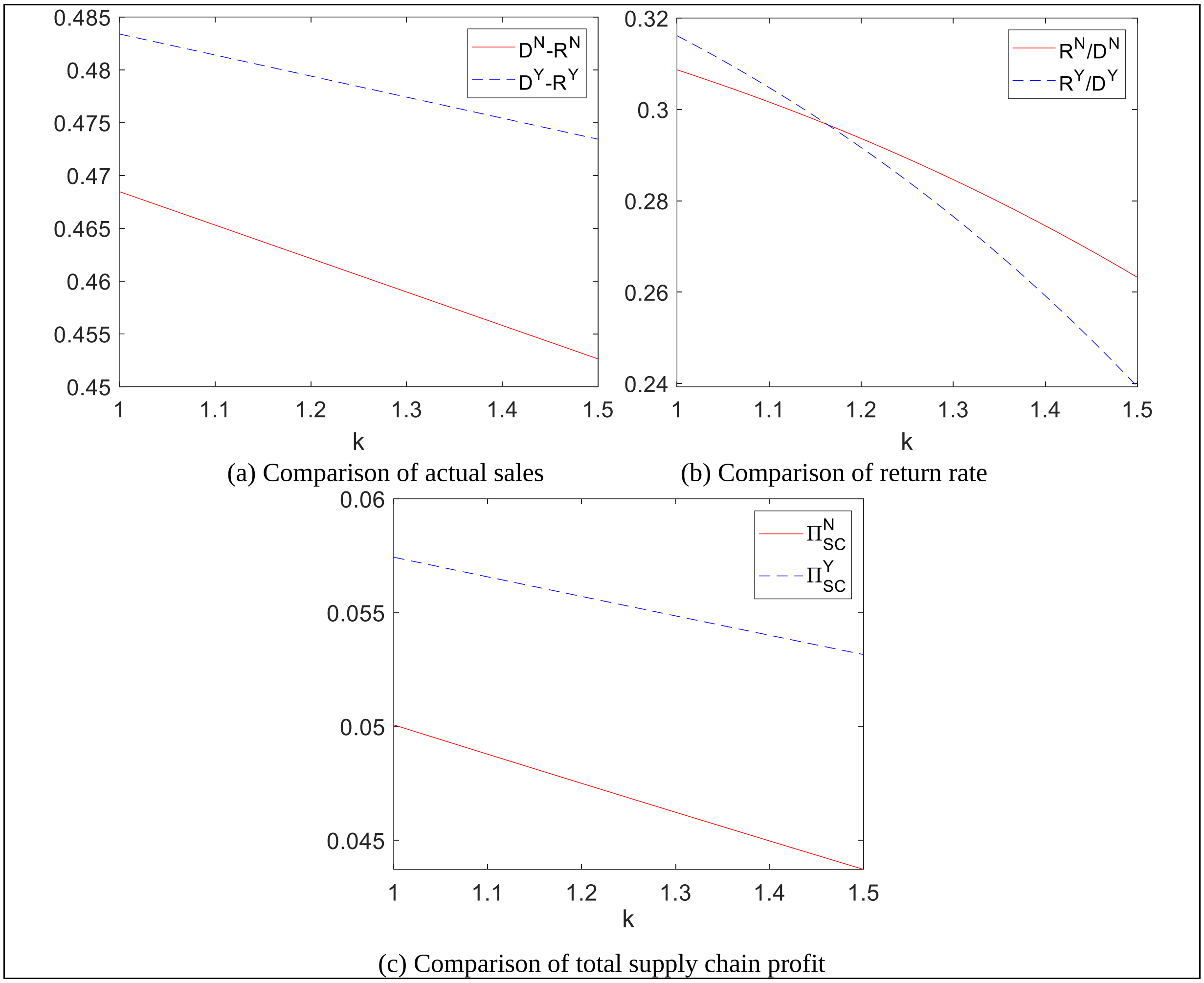


(a) Comparison of actual sales

(b) Comparison of return rate

(c) Comparison of total supply chain profit

Figure 7: Comparison of Model Y and Model N when k<1



(a) Comparison of actual sales (b) Comparison of return rate

(c) Comparison of total supply chain profit

Figure 8: Comparison of Model Y and Model N when k>1

From Figures 7 and 8, we see that whether or not the actual quality is higher than the expected quality, quality cooperation can increase the actual sales and total profit, but it does not necessarily reduce the return rate. In addition, from Figure 8(b), we conclude that when the actual quality is less than the expected quality by a relatively large margin, the brand manufacturer and influencer can effectively reduce product returns through quality cooperation.

6.3. Joint effort of quality improvement

In the basic model outlined in Section 3.3, the influencer indirectly affects product quality through quality cost sharing. However, in order to more intuitively reflect the impact of the influencer on product quality, we follow Zhou et al. (2022) by considering that both the brand manufacturer and influencer make efforts to improve product quality. In this extended model, the influencer's quality effort directly affects product quality through their active involvement in product design and testing. For example, Oshiman, a domestic skincare brand, collaborates with Jiaqi Li based on fan data to create a water and milk set for launch in the live streaming room. As such, we assume that the product quality level q is affected by the quality effort levels of the brand manufacturer and influencer q_1 and q_2, i.e., $q = q_1 + q_2$. Substituting $q = q_1 + q_2$ into Eqs (1) and (2), we obtain the functions of demand and return volume as follows:

$$D = \theta A \int_{\alpha p-\mu e-\lambda Q+r}^{1} dv + (1-\theta) A \int_{\alpha p-\lambda Q+r}^{1} dv = A(1-\alpha p+\theta\mu e+\lambda Q-r),$$

$$R = \theta A \int_{\alpha p-\mu e-\lambda Q+r}^{\frac{\alpha p-\mu e-\lambda(q_1+q_2)-r}{\tau}} dv + (1-\theta) A \int_{\alpha p-\lambda Q+r}^{\frac{\alpha p-\lambda(q_1+q_2)-r}{\tau}} dv$$

$$= A\left(\frac{\alpha p-\theta\mu e-\lambda(q_1+q_2)-r}{\tau} - \alpha p+\theta\mu e+\lambda Q-r\right).$$

When the influencer participates in quality cooperation, the functions of the brand manufacturer's and influencer's profits are as follows:

$$\Pi_M^Y = ((1-\rho)p - c_0)(D-R) - c_1 R - \sigma {q_1}^2 - F,$$
$$\Pi_L^Y = \rho p(D-R) - \xi e^2 - \sigma {q_2}^2 + F.$$

The decision sequence of the model is as follows: First, the brand manufacturer and influencer simultaneously decide the quality effort levels q_1 and q_2, respectively; then, the influencer decides the marketing effort level e. We derive the optimal solution using backward induction as follows:

$$e^{Y*} = \frac{A\theta\mu\rho p}{2\xi\tau},\ {q_1}^{Y*} = \frac{A\lambda(p(1-\rho)-c_0+c_1)}{2\sigma\tau},\ {q_2}^{Y*} = \frac{A\lambda\rho p}{2\sigma\tau}.$$

Substituting $q = q_1 + q_2$ into Eqs (1) and (2), we derive the functions of demand and return volume as follows:

$$D^{Y*} = \frac{A\left(Ap\theta^2\mu^2\rho + 2(1-r-p\alpha+Q\lambda)\xi\tau\right)}{2\xi\tau},$$
$$R^{Y*} = \frac{A(-Ap(\lambda^2\xi+\theta^2\mu^2\rho\sigma(1-\tau)) + 2\xi\sigma\tau(p\alpha(1-\tau)+Q\lambda\tau-r(1+\tau)) + A\lambda^2\xi(c_0-c_1))}{2\xi\sigma\tau^2},$$
$$\Pi_M^{Y*} = \frac{A(4\xi\sigma\tau I_5 + Ap\lambda^2\xi I_4 + 2Ap\theta^2\mu^2\rho\sigma I_2 + A\lambda^2\xi I_6)}{4\xi\sigma\tau^2} - F,$$
$$\Pi_L^{Y*} = \frac{Ap\rho(Ap(\beta^2\lambda^2\xi(2-\rho)+\theta^2\mu^2\rho\sigma) - 4\xi\sigma\tau(r+p\alpha-\tau) - 2A\beta^2\lambda^2\xi(c_0-c_1))}{4\xi\sigma\tau^2} + F,$$

where $I_4 = (1+\rho)p(1-\rho) - 2c_0 + 2c_1$, $I_5 = (r+p\alpha-\tau)(p(-1+\rho)+c_0) + (p\alpha(-1+\tau) - Q\lambda\tau + r(1+\tau))c_1$, and $I_6 = (c_0 - c_1)^2$.

Next, we compare the model with quality cooperation and the model without quality cooperation to derive the following result.

Proposition 6. In this extended model, quality cooperation can improve product quality and actual sales, and reduce return rate, i.e., $q^{N*} < q^{Y*}$, $D^{N*} - R^{N*} < D^{Y*} - R^{Y*}$, and $\frac{R^{N*}}{D^{N*}} > \frac{R^{Y*}}{D^{Y*}}$. Besides, quality cooperation can increase the influencer's profit but decrease the brand manufacturer's profit, i.e., $\pi_M^{N*} > \pi_M^{Y*}$ and $\pi_L^{N*} < \pi_L^{Y*}$.

From Proposition 6, quality cooperation has several benefits, including improving product quality, increasing actual sales, and reducing return rate. These findings are consistent with Proposition 4. In addition, quality cooperation increases the influencer's profit but decreases the brand manufacturer's profit, which is inconsistent with Proposition 5 that "quality cooperation can increase profits for both parties involved". This discrepancy indicates that when the influencer directly influences product quality, the brand manufacturer loses its independent decision-making power regarding product production, and research and development. The influencer's ability to influence marketing and quality simultaneously gives them the incentive to pursue higher profits. Unfortunately, this pursuit of greater profits comes at the expense of the brand manufacturer's profit. As a result, this cooperative strategy is not conducive to maintaining a stable cooperative relationship between the brand manufacturer and influencer.

6.4. The long-term cooperation versus short-term cooperation

Due to the increase in quality investment cost in the case of quality cooperation, the brand manufacturer may consider opening its own live streaming room to reduce cost after cooperating with the influencer. However, the brand manufacturer lacks live streaming e-commerce expertise, which runs the risk of unmarketable sales. Therefore, this section expands the model from a single sales period to two sales periods to explore whether the brand manufacturer and influencer should choose the long-term cooperation strategy, i.e. cooperate with each other in the second period.

Under the two-period model, consumers' demand function, return function, and profit function change as follows:

Market demand. In the first period, the expected utility of consumers is the same as that in the single sales cycle, regardless of whether or not the two parties choose the long-term cooperation strategy. Therefore, the expected utility of the followers and non-followers are $U_{l1} = \nu - \alpha p + \lambda Q + \mu e - r$ and $U_{o1} = \nu - \alpha p + \lambda Q - r$, respectively. In the second period, if the brand manufacturer chooses to open its own live streaming room, i.e., the short-term cooperation strategy, the influencer's marketing efforts in the first period will affect the sales in the second period, which means that live streaming e-commerce has a spillover effect b(0<b<1). Therefore, under short-term cooperation, the expected utility of the followers and non-followers in the second period are $U_{l2}^S = \nu - \alpha p + \lambda q + b\mu e$ and $U_{o2}^S = \nu - \alpha p + \lambda$, respectively. Under long-term cooperation, the expected utility of the followers and non-followers in the second period are $U_{l2}^L = \nu - \alpha p + \lambda q + \mu e$ and $U_{o2}^L = \nu - \alpha p + \lambda q$, respectively. Based on the above analysis, consumers' demand in the first period is

$$D_1 = \theta A \int_{\alpha p - \mu e - \lambda Q + r}^{1} dv + (1-\theta) A \int_{\alpha p - \lambda Q + r}^{1} dv = A(1 - \alpha p + \theta\mu e + \lambda Q - r). \quad (15)$$

Under short-term cooperation, consumers' demand in the second period is

$$D_2^s = \theta A \int_{\alpha p-\lambda q-b\mu e}^{1} 1\, dv + (1-\theta) A \int_{\alpha p-\lambda q}^{1} 1\, dv = A(1-\alpha p+\lambda q+\theta b\mu e). \tag{16}$$

Under long-term cooperation, consumers' demand in the second period is

$$D_2^L = \theta A \int_{\alpha p-\lambda q-\mu e}^{1} 1\, dv + (1-\theta) A \int_{\alpha p-\lambda q}^{1} 1\, dv = A(1-\alpha p+\lambda q+\theta \mu e). \tag{17}$$

Consumer returns. Consumers' return behaviour is similarly affected by the long delivery time and return hassle cost. Therefore, the actual utility of the followers and non-followers in the first period are $U_{l1}' = \tau v - \alpha p + \lambda q + \mu e$ and $U_{o1}' = \tau v - \alpha p + \lambda q$, respectively. Under short-term cooperation, the actual utility of the followers and non-followers in the second period are ${U_{l2}^S}' = \tau v - \alpha p + \lambda q + b\mu e$ and ${U_{o2}^S}' = \tau v - \alpha p + \lambda q$, respectively. Under long-term cooperation, the actual utility of the followers and non-followers in the second period are ${U_{l2}^L}' = \tau v - \alpha p + \lambda q + \mu e$ and ${U_{o2}^L}' = \tau v - \alpha p + \lambda q$, respectively. Based on the above analysis, the total amount of return in the first period is

$$R_1 = \theta A \int_{\alpha p-\mu e-\lambda Q+r}^{\frac{\alpha p-\mu e-\lambda q-r}{\tau}} dv + (1-\theta) A \int_{\alpha p-\lambda Q+r}^{\frac{\alpha p-\lambda q-r}{\tau}} dv = A\left(\frac{\alpha p-\theta\mu e-\lambda q-r}{\tau} - \alpha p + \theta\mu e + \lambda Q - r\right). \tag{18}$$

Under short-term cooperation, the total amount of return in the second period is

$$R_2^S = \theta A \int_{\alpha p-\lambda q-b\mu e}^{\frac{\alpha p-\lambda q-b\mu e-r}{\tau}} 1\, dv + (1-\theta) A \int_{\alpha p-\lambda q}^{\frac{\alpha p-\lambda q-r}{\tau}} 1\, dv = A\left(\frac{\alpha p-\theta b\mu e-\lambda q-r}{\tau} - \alpha p + \theta b\mu e + \lambda q\right). \tag{19}$$

Under long-term cooperation, the total amount of return in the second period is

$$R_2^L = \theta A \int_{\alpha p-\lambda q-\mu e}^{\frac{\alpha p-\lambda q-\mu e-r}{\tau}} 1\, dv + (1-\theta) A \int_{\alpha p-\lambda q}^{\frac{\alpha p-\lambda q-r}{\tau}} 1\, dv = A\left(\frac{\alpha p-\theta\mu e-\lambda q-r}{\tau} - \alpha p + \theta\mu e + \lambda q\right). \tag{20}$$

Supply Chain Profit. In the first period, the brand manufacturer and influencer bear the quality investment cost incurred from the research and design of new products. In the second period, the brand manufacturer only bears the production cost and return handling cost, and the influencer only bears the marketing investment cost. Therefore, under short-term cooperation, the profits of the brand manufacturer and influencer in the first period are

$$\Pi_{\mathrm{M1}}^{SY} = ((1-\rho)p - c_0)(D_1 - R_1) - c_1 R_1 - (1-\phi)\sigma q^2 - F, \tag{21}$$

$$\Pi_{\mathrm{L1}}^{SY} = \rho p (D_1 - R_1) - \xi e^2 - \phi\sigma q^2 + F. \tag{22}$$

The profits for the brand manufacturer and influencer in the second period are

$$\Pi_{\mathrm{M2}}^{SY} = (p - c_0)(D_2^s - R_2^s) - c_1 R_2^s. \tag{23}$$

The total profits for the brand manufacturer and influencer are

$$\begin{aligned}\Pi_M^{SY} = \Pi_{\mathrm{M1}}^{SY} + \Pi_{\mathrm{M2}}^{SY} &= \big((1-\rho)p - c_0\big)(D_1 - R_1) + (p - c_0)(D_2^s - R_2^s) \\ &\quad - c_1(R_2^s + R_1) - (1-\phi)\sigma q^2 - F,\end{aligned} \tag{24}$$

$$\Pi_L^{SY} = \Pi_{\mathrm{L1}}^{SY} = \rho p (D_1 - R_1) - \xi e^2 - \phi\sigma q^2 + F. \tag{25}$$

Under long-term cooperation, the profits for the brand manufacturer and influencer in the first period are

$$\Pi_{\mathrm{M1}}^{LY} = ((1-\rho)p - c_0)(D_1 - R_1) - c_1 R_1 - (1-\phi)\sigma q^2 - F, \tag{26}$$

$$\Pi_{\mathrm{L1}}^{LY} = \rho p (D_1 - R_1) - \xi e^2 - \phi\sigma q^2 + F. \tag{27}$$

The profits for the brand manufacturer and influencer in the second period are

$$\Pi_{M2}^{LY} = ((1-\rho)p - c_0)(D_2^L - R_2^L) - c_1 R_2^L - F, \tag{28}$$

$$\Pi_{L2}^{LY} = \rho p (D_2^L - R_2^L) - \xi e^2 + F. \tag{29}$$

The total profits of the brand manufacturer and influencer are

$$\begin{aligned}\Pi_M^{LY} = \Pi_{\mathrm{M1}}^{LY} + \Pi_{\mathrm{M2}}^{LY} &= \big((1-\rho)p - c_0\big)(D_1 - R_1 + D_2^L - R_2^L) \\ &\quad - c_1(R_1 + R_2^L) - (1-\phi)\sigma q^2 - 2F,\end{aligned} \tag{30}$$

$$\Pi_L^{LY} = \Pi_{\mathrm{L1}}^{LY} = \rho p (D_1 - R_1 + D_2^L - R_2^L) - \xi e^2 - \phi\sigma q^2 + 2F. \tag{31}$$

Using backward induction, we find the optimal solutions under short-term cooperation as

$$p^{SY*} = \frac{\begin{pmatrix} 2A\lambda^2\xi(2-\rho)\tau(c_0-(2-\tau)c_1) - 2\sigma\tau(1-\phi)(2\xi(2-\rho)\tau(r+\tau) \\ +(2\alpha\xi\tau - A\theta^2\mu^2\rho)c_0 - (-1+\tau)(A(1+b)\theta^2\mu^2\rho - 4\alpha\xi\tau)c_1) \end{pmatrix}}{2\tau(A\lambda^2\xi(2-\rho)^2 + 2\sigma(1-\phi)(A\theta^2\mu^2(1+b-\rho)\rho - 2\alpha\xi(2-\rho)\tau))},$$

$$q^{SY*} = \frac{\begin{pmatrix} A\lambda(2\xi(2-\rho)^2\tau(r+\tau) + (A\theta^2\mu^2(2b-\rho)\rho - 2\alpha\xi(2-\rho)\tau)c_0 \\ +(4\alpha\xi(2-\rho)\tau + A\theta^2\mu^2\rho(-2+3\rho-b(2+\rho)-(1-b)\rho\tau))c_1) \end{pmatrix}}{-2\tau(A\lambda^2\xi(2-\rho)^2 + 2\sigma(1-\phi)(A\theta^2\mu^2(1+b-\rho)\rho - 2\alpha\xi(2-\rho)\tau))},$$

$$e^{SY*} = \frac{A\theta\mu\rho\begin{pmatrix} 2A\lambda^2\xi(2-\rho)\tau(c_0-(2-\tau)c_1) - 2\sigma\tau(1-\phi)(2\xi(2-\rho)\tau(r+\tau) \\ +(2\alpha\xi\tau - A\theta^2\mu^2\rho)c_0 + (1-\tau)(A(1+b)\theta^2\mu^2\rho - 4\alpha\xi\tau)c_1) \end{pmatrix}}{4\xi\tau^2(A\lambda^2\xi(2-\rho)^2 + 2A\theta^2\mu^2(1+b-\rho)\rho\sigma - 4\alpha\xi(2-\rho)\sigma\tau)},$$

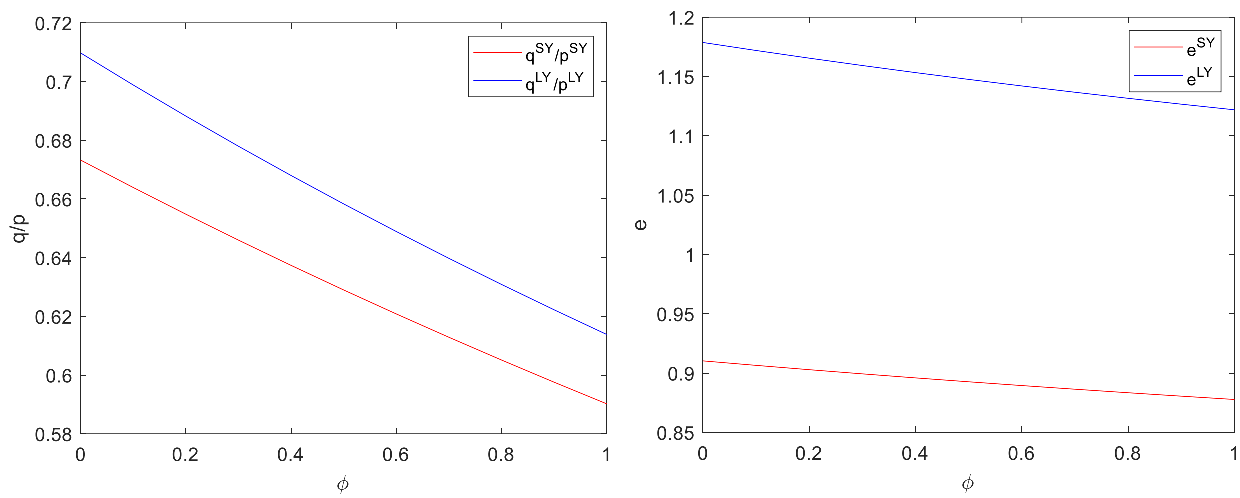
and the optimal solutions under long-term cooperation as

$$p^{LY*} = \frac{\begin{pmatrix} 2A\lambda^2\xi(1-\rho)\tau(2c_0-(2-\tau)c_1) - 2\sigma\tau(1-\phi)(2\xi(1-\rho)\tau(r+\tau) \\ -(A\theta^2\mu^2\rho - 2\alpha\xi\tau)(c_0-(1-\tau)c_1)) \end{pmatrix}}{4(1-\rho)\tau(A\lambda^2\xi(1-\rho) + \sigma(A\theta^2\mu^2\rho - 2\alpha\xi\tau)(1-\phi))},$$

$$q^{LY*}=\frac{A\lambda(-2\xi(1-\rho)\tau(r+\tau)-(A\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-c_1))}{2\tau(A\lambda^2\xi(1-\rho)+\sigma(A\theta^2\mu^2\rho-2\alpha\xi\tau)(1-\phi))},$$

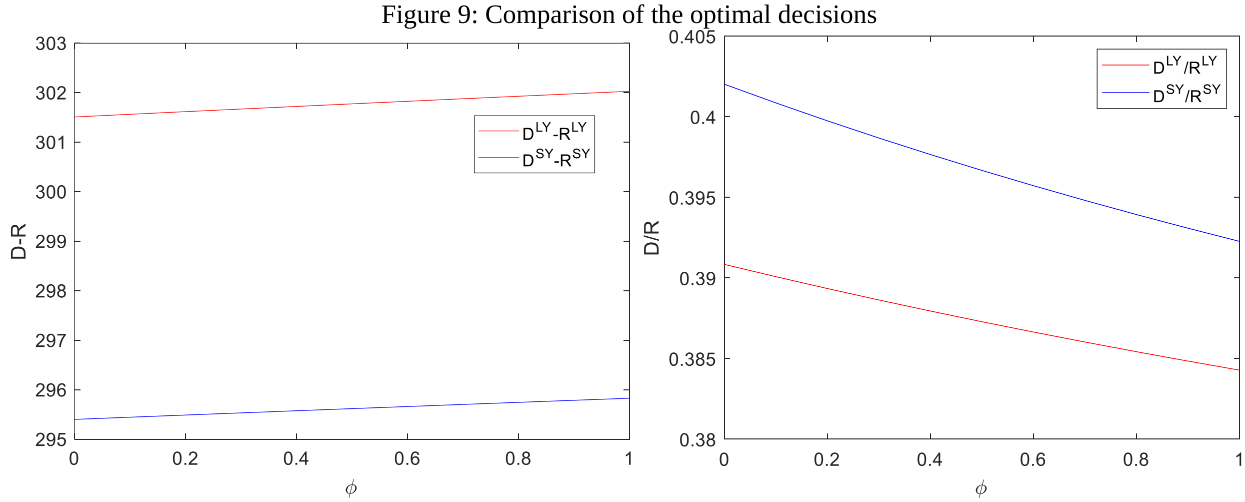
$$e^{LY*}=\frac{A\theta\mu\rho\begin{pmatrix}2A\lambda^2\xi(1-\rho)\tau(2c_0-(2-\tau)c_1)-2\sigma\tau(1-\phi)(2\xi(1-\rho)\tau(r+\tau)\\-(A\theta^2\mu^2\rho-2\alpha\xi\tau)(c_0-(1-\tau)c_1))\end{pmatrix}}{4\xi(1-\rho)\tau^2(A\lambda^2\xi(1-\rho)+\sigma(A\theta^2\mu^2\rho-2\alpha\xi\tau)(1-\phi))}.$$

Next, we conduct numerical studies to compare the optimal decisions, return rates, actual sales, and profits of short-term cooperation and long-term cooperation. Setting A=20, α=1.5, λ=1, μ=0.5, θ=0.5, Q=10, r=0.01, τ=0.95, c_0=1, c_1=0.01, σ=1, ξ=1, F=3, ρ=0.3, b=0.3, and $\phi\in$[0,1], we obtain the results shown in Figures 9 and 10.



(a) Comparison of quality-price ratio (b) Comparison of marketing level

Figure 9: Comparison of the optimal decisions



(a) Comparison of actual sales (b) Comparison of return rate

Figure 10: Comparison of actual sales and return rate

From Figure 9, we see that the price, quality level, and influencer's marketing level are higher under long-term cooperation than under short-term cooperation. Besides, under long-term cooperation, the brand manufacturer can provide higher quality-price-ratio products. From Figure 10, we see that long-term cooperation can effectively increase the actual sales of the product and reduce the return rate. This also shows that long-term cooperation is conducive to increasing brand awareness and promoting product upgrading.

In addition, we set $b=0.3, 0.6$ to compare the brand manufacturer's and influencer's profits, as well as the difference of supply chain profit respectively. We find that long-term cooperation increases the influencer's profit, while the change in the brand manufacturer's profit is related to the live streaming spillover effect. When the spillover effect is larger, the brand manufacturer's profit under long-term cooperation is smaller than that under short-term cooperation. However, when the spillover effect is smaller, with increasing proportion of the influencer's quality investment, the brand manufacturer's profit under long-term cooperation is gradually close to that under short-term cooperation, or even lower than that under short-term cooperation. From the perspective of the supply chain, the long-term cooperation strategy can improve the supply chain profit when the influencer participates in quality cooperation. In addition, when the spillover effect is larger, the profit gap between short-term cooperation and long-term cooperation is smaller.

7. Conclusion, managerial insights and future research

7.1. Concluding remarks

Live streaming as a new channel to sell products has promoted consumers to purchase products with the advantage of the discounted price and attractive marketing ways. However, due to product quality uncertainty of online shopping, the substandard product quality, i.e., the actual quality of the product does not reach the expected quality, may cause consumers' return behaviour. To reduce return rate, some influencers have cooperated with brand manufacturers in product quality to improve product quality. However, how the cooperation between the influencer and the brand manufacturer can be successful is an interesting problem to research. We hence develop a game-theoretic model and analyze the impact of cooperation on the optimal decisions of influencer and brand manufacture. We intend to expand the research on live streaming e-commerce and provide relevant managerial insights to guide the managers to pursue cooperation more smoothly.

7.2. Managerial insights and implications

We generate some important managerial insights and implications as follows:

7.2.1 Effect of quality cooperation

In our study we explore the importance and effect of quality cooperation. We discuss the implications of our research findings as follows:

(i) Factors that affect quality cooperation: The success of quality cooperation mainly depends on the return hassle cost, commission rate, blanket fee, and percentage of the influencer's quality investment. The influencer should not set a relatively high commission rate and blanket fee, while the brand manufacturer should provide a convenient return service to reduce consumers' return hassle cost. Moreover, the percentage of the influencer's quality investment should not be excessive large. Otherwise, the brand manufacturer and influencer cannot engage in quality cooperation smoothly.

(ii) Impacts of quality cooperation on actual sales and return rate: When the influencer participates in quality cooperation, the product quality and actual sales increase, and the return rate decreases, but the demand for the product may decrease. Hence, the actual sales of the product should be regarded as a reference basis when signing a cooperation contract. In reality, to secure the brand manufacturer's interest, some influencers make minimum sales commitments. In addition, it is not a long-term solution to attract flows through a variety of marketing strategies in live streaming e-commerce. For brand manufacturers and influencers, focusing on product quality is the most effective way to address the issue of a high return rate.

(iii) Impacts of quality cooperation on the profits of the brand manufacturer and influencer: When the influencer participates in a quality cooperation in a cost sharing way, quality cooperation can increase the profits of both the brand manufacturer and influencer if the percentage of the influencer's quality investment and commission rate are in proper ranges. However, when the influencer directly affects the product quality level through their own quality effort, quality cooperation can only increase the influencer's profit. This indicates that if the influencer is involved in the production and design of the product and directly affects the product quality level, the influencer will gradually be in a dominant role in the live streaming e-commerce industry, which will be detrimental to the development of the brand manufacturer in the live streaming e-commerce channel.

7.2.2 Impacts of consumers' sensitivity to product quality

We analyze the impacts of consumers' sensitivity to product quality on the profits of the brand manufacturer and influencer under two situations. We summarize the managerial insights as follows:

(i) Profit of the influencer: Increasing consumers' sensitivity to product quality is not necessarily beneficial to the influencer. In the situation without quality cooperation, consumers' sensitivity to product quality positively affects the influencer's profit. However, changes in the profit of the influencer are related to the commission rate. Increasing consumers' sensitivity to product quality will increase the influencer's profit only when the commission rate is relatively high. For the influencer, if consumers become more sensitive to product quality, participating in quality cooperation and raising the commission rate can improve their profit. This indicates that increasing consumers' requirements for product quality can force the influencer to pay more attention to product quality. The influencer should control the quality of the products that they introduce to maintain their good reputation and attract more consumers.

(ii) Profit of the brand manufacturer: No matter which cooperation strategy is chosen, when the coefficient of quality investment is relatively low, increasing consumers' sensitivity to product quality can lead to more profit for the manufacturer. This requires the brand manufacturer to optimize its production process as much as possible, while focusing on improving product quality, thus increasing production efficiency and reducing production cost.

7.2.3 Impacts of the proportion of followers

Considering differences in consumers' attitudes towards the influencer's marketing way, we analyze the impacts of the proportion of followers on the profits of the brand manufacturer and influencer. The relevant conclusions and implications are as follows:

(i) Profit of the influencer: When the potential market demand is relatively large and the commission rate is relatively low, the larger the proportion of followers is, the higher is the profit for the influencer. This means that the influencer with more followers does not necessarily have a greater ability to introduce products. Therefore, the influencer should choose products that well match their image and perform precise marketing for the products to achieve better performance. This explains why some stars fail to sell products through live streaming.

(ii) Profit of the brand manufacturer: When the price is constant, increasing the proportion of followers will not affect product quality but will increase the return volume. If the commission rate is relatively high, the increment of the brand manufacturer's revenue will be lower than that of its cost, thus leading to a lower profit. When the price is adjustable, increasing the proportion of followers can prompt the brand manufacturer to provide products with higher prices and quality, so improving the brand manufacturer's profit. For the brand manufacturer, when the influencer's followers increase, they could upgrade the products and provide high-value products for consumers. In addition, due to the increase in sales, the brand manufacturer could try to negotiate with the influencer to reduce the commission rate, thus maintaining its profit and promoting long-term cooperation between the two parties.

7.2.4 Comparisons of long-term versus short-term cooperation

Considering the brand manufacture may choose to opening its own live streaming room after cooperating with the influencer, we explore the choices of long-term versus short-term cooperation. We find the following managerial insights:

(i) ***The effect of long-term cooperation:*** Long-term cooperation can promote product upgrading, increase product sales, and reduce the return rate, thus gathering more loyal consumers for the brand. Besides, long-term cooperation can improve the overall performance of the supply chain and promote high-quality development of live e-commerce.

(ii) Choices between long-term and short-term cooperation: When the spillover effect is smaller, the brand manufacturer should choose short-term cooperation with the influencer for product credit endorsement in the first period. When the spillover effect is smaller, if the willingness of the influencer to participate in quality cooperation is relatively low, the brand manufacturer should choose long-term cooperation.

7.3. Future studies

All in all, there are some limitations in our model, which point to interesting directions for future research. First, we consider the brand manufacturer as the leader in the Stackelberg game, but some influencer may play a dominant role in the market. Thus, future research can take the influencer as the leader and compare the optimal decisions under different power structures of the players. Second, we mainly focus on a single live streaming channel in our study. Noting that the sales of the live streaming channel may impact the other channels' sales, future research can extend our work by considering the multi-channel supply chain. Third, a three-party game including a brand manufacturer, an influencer, and an e-commerce platform is worth studying. Fourth, we assume that the time discount factor tau is a constant to simplify solution of the problems under study. However, assuming tau as a random variable can address more complex problems, which is another important topic for future research. Fifth, we assume that all the consumers will consider the return hassle cost. However, some consumers do not know if they will keep or return the product before receiving the product. Hence, future research may consider the return hassle cost as an expected value to produce more interesting results. Finally, the multi-period setting from the long-term decision-making perspective can be considered.

Acknowledgment

This research was supported by the Humanities and Social Sciences Youth Foundation, Ministry of Education (No. 23YJC630204), National Natural Science Foundation of China (No. 71831001), R&D Program of Beijing Municipal Education Commission (No. SM202310011001), and National Natural Science Foundation of Beijing (No. 9222007).

REFERENCES

Alaei, A. M., Taleizadeh A. A., & Rabbani. M. (2022). Marketplace, reseller, or web-store channel: the impact of return policy and cross-channel spillover from marketplace to web-store. *Journal of Retailing and Consumer Services*, *65*, 102271.

Alolayyan, M. N., Mohammed, J., Quoquab, F., & Thurasamy, R. (2020). The effect of user-generated content quality on brand engagement: the mediating role of functional and emotional values. *Journal of Electronic Commerce Research*, *21*(1), 39-55.

Aksoy, N. C., Kabadayi, E. T., Yilmaz, C., & Alan, A. K. (2023). Personalization in Marketing: How do people perceive personalization practices in the business world? *Journal of Electronic Commerce Research*, *24*(4), 269-297.

Ambilkar, P., Dohale V., Gunasekaran A., & Bilolikar V. (2022). Product Returns Management: A comprehensive review and future research agenda. *International Journal of Production Research*, *60*(12), 3920-3944.

Borenich, A., Dickbauer Y., Reimann M., & Souza G.C. (2020). Should a manufacturer sell refurbished returns on the secondary market to incentivize retailers to reduce consumer returns? *European Journal of Operational Research*, *282*(2), 569-579.

Cao, K., & Choi T.M. (2022). Optimal Trade-in Return Policies: Is it wise to be generous?. *Production and Operations Management*, *31*(3), 1309-1331.

Chakraborty, T., Chauhan, S. S., & Ouhimmou, M. (2019). Cost-sharing mechanism for product quality improvement in a supply chain under competition. *International Journal of Production Economics*, *208,* 566-587.

Chen, J., & Chen B. (2016). Competing with customer returns policies. *International Journal of Production Research*, *54*(7), 2093-2107.

Chen, L., Nan G., & Li M. (2018). Wholesale pricing or agency pricing on online retail platforms: the effects of customer loyalty. *International Journal of Electronic Commerce*, *22*(4), 576-608.

Chia, K. C., Hsu, C. C., Lin, L. T., & Tseng, H. H. (2021). The identification of ideal social media influencers: integrating the social capital, social exchange, and social learning theories. *Journal of Electronic Commerce Research*, *22*(1), 4-21.

Clement Addo, P., Fang, J., Asare, A. O., & Kulbo, N. B. (2021). Customer engagement and purchase intention in live-streaming digital marketing platforms. *The Service Industries Journal*, *41*(11-12), 767-786.

De Veirman, M., Cauberghe V., & Hudders L. (2017). Marketing through instagram influencers: the impact of number of followers and product divergence on brand attitude. *International Journal of Advertising*, *36*(5), 798-828.

Dhun, & Dangi, H. K. (2023). Influencer marketing: Role of influencer credibility and congruence on brand attitude and eWOM. *Journal of Internet Commerce*, *22*(sup1), S28-S72.

Ertekin, N., & Agrawal, A. (2021). How does a return period policy change affect multichannel retailer profitability?. *Manufacturing & Service Operations Management*, *23*(1), 210-229.

Fan, T., Wang L., & Song Y. (2022). Impact of live commerce spillover effect on supply chain decisions. *Industrial Management & Data Systems*, *122*(4), 1109-1127.

Fan, Z. P., & Chen, Z. (2020). When should the e-tailer offer complimentary return-freight insurance?. *International Journal of Production Economics*, *230*, 107890.

Farivar, S., Wang, F., & Turel, O. (2022). Followers' problematic engagement with influencers on social media: An attachment theory perspective. *Computers in Human Behavior*, *133*, 107288.

Fu, X., Liu, S., Fang, B., Luo, X., & Cai, S. (2020). How do expectations shape consumer satisfaction? An empirical study on knowledge products. *Journal of Electronic Commerce Research*, *21*(1), 1-20.

Ghosh, D., & Shah, J. (2015). Supply chain analysis under green sensitive consumer demand and cost sharing contract. *International Journal of Production Economics*, *164*, 319-329.

Hardisty, D.J., & Pfeffer J. (2017). Intertemporal uncertainty avoidance: when the future is uncertain, people prefer the present, and when the present is uncertain, people prefer the future. *Management Science*, 63(2), 519-527.

He, H., Wang, C., Wang, S., Ma, F., Sun, Q., & Zhao, X. (2021). Does environmental concern promote EV sales? Duopoly pricing analysis considering consumer heterogeneity. *Transportation Research Part D: Transport and Environment*, *91*, 102695.

Heydari, J., Bineshpour, P., Walther, G., & Ülkü, M. A. (2022). Reconciling conflict of interests in a green retailing channel with green sales effort. *Journal of Retailing and Consumer Services*, *64*, 102752.

Heydari, J., Govindan K., & Basiri Z. (2021). Balancing price and green quality in presence of consumer environmental awareness: A green supply chain coordination approach. *International Journal of Production Research*, *59*(7), 1957-1975.

Hu, X., Wan Z., & Murthy N.N. (2019). Dynamic pricing of limited inventories with product returns. *M&som-Manufacturing & Service Operations Management*, *21*(3), 501-518.

Hua, Y., Bao, L., & Wu, X. (2021). The product-selling strategy under direct and indirect value identification. *Journal of cleaner production*, *279*, 123591.

Hübner, A., Holzapfel A., & Kuhn H. (2016). Distribution systems in omni-channel retailing. *Business Research*, *9* (2), 255-296.

Li, G., Li L., Sethi S.P., & Guan X. (2019). Return strategy and pricing in a dual-channel supply chain. *International Journal of Production Economics*, *215*, 153–164.

Li, M., & Liu Y. (2021). Beneficial product returns in supply chains. *Production and Operations Management*, *30* (11), 3849-3855.

Li, W., & Chen, J. (2020). Manufacturer's vertical integration strategies in a three-tier supply chain. *Transportation Research Part E: Logistics and Transportation Review*, *135*, 101884.

Lin, J., Zhang, J., & Cheng, T. C. E. (2020). Optimal pricing and return policy and the value of freight insurance for a retailer facing heterogeneous consumers with uncertain product values. *International Journal of Production Economics*, *229*, 107767.

Liu, J., Xu, Q., & Sun, Z. (2022). Joint optimization decision of online retailers' pricing and live-streaming effort in the postepidemic era. *Complexity*, *2022*, 1-11.

Liu, S., Hua G., Cheng T.C.E., & Choi T.-M. (2022). Optimal pricing and quality decisions in supply chains with consumers' anticipated regret and online celebrity retailers. *IEEE Transactions on Engineering Management*, 1-15.

Liu, X., Wang, D., Gu, M., & Yang, J. (2023). Research on the influence mechanism of anchors' professionalism on consumers' impulse buying intention in the livestream shopping scenario. *Enterprise Information Systems*, *17*(7), 2065457.

Loewenstein, G. F. (1988). Frames of mind in intertemporal choice. *Management science*, *34*(2), 200-214.

Lu, B., & Chen Z. (2021). Live streaming commerce and consumers' purchase intention: an uncertainty reduction perspective. *Information & Management*, *58*(7), 103509.

Luo, L., & Sun J. (2016). New product design under channel acceptance: Brick-and-Mortar, Online-Exclusive, or Brick-and-Click. *Production and Operations Management*, *25*(12), 2014-2034.

Lv, H., & Li, D. (2021). Impacts of heterogeneous green consumers on green innovation in electric vehicle and charging pile firms. *Sustainable Production and Consumption*, *28*, 1216-1231.

Mao, Z., Du, Z., Yuan, R., & Miao, Q. (2022). Short-term or long-term cooperation between retailer and MCN? New launched products sales strategies in live streaming e-commerce. *Journal of Retailing and Consumer Services*, *67*, 102996.

Meng, Q., Wang Y., Zhang Z., & He Y. (2021a). Supply chain green innovation subsidy strategy considering consumer heterogeneity. *Journal of Cleaner Production*, *281*, 125199.

Meng, Q., Li M., Liu W., Li Z., & Zhang J. (2021b). Pricing policies of dual-channel green supply chain: considering government subsidies and consumers' dual preferences. *Sustainable Production and Consumption*, *26*, 1021-1030.

Mukhopadhyay, S. K., & Setaputra R. (2007). A dynamic model for optimal design quality and return policies. *European Journal of Operational Research*, *180*(3), 1144-1154.

Najafi-Ghobadi, S., Bagherinejad, J., & Taleizadeh, A. A. (2021). Modeling the diffusion of generation products in the presence of heterogeneous strategic customers for determining optimal marketing-mix strategies. *Computers & Industrial Engineering*, *160*, 107606.

Radhi, M., & Zhang, G. (2018). Pricing policies for a dual-channel retailer with cross-channel returns. *Computers & Industrial Engineering*, *119*, 63-75.

Ren, M., Liu, J., Feng, S., & Yang, A. (2021). Pricing and return strategy of online retailers based on return insurance. *Journal of Retailing and Consumer Services*, *59*, 102350.

Sadeghi, R., Taleizadeh, A. A., Chan, F. T., & Heydari, J. (2019). Coordinating and pricing decisions in two competitive reverse supply chains with different channel structures. *International Journal of Production Research*, *57*(9), 2601-2625.

Sarkar, S., & Bhadouriya, A. (2020). Manufacturer competition and collusion in a two-echelon green supply chain with production trade-off between non-green and green quality. *Journal of cleaner production*, *253*, 119904.

Sun, Y., Shao, X., Li, X., Guo, Y., & Nie, K. (2020). A 2020 perspective on "How live streaming influences purchase intentions in social commerce: An IT affordance perspective". *Electronic Commerce Research and Applications*, *40*, 100958.

Taleizadeh, A. A., Beydokhti, S. R., Cárdenas-Barrón, L. E., & Najafi-Ghobadi, S. (2021). Pricing of complementary products in online purchasing under return policy. *Journal of Theoretical and Applied Electronic Commerce Research*, *16*(5), 1718-1739.

Taleizadeh, A.A., Noori-Daryan M., & Sana S.S. (2020). Manufacturing and selling tactics for a green supply chain under a green cost sharing and a refund agreement. *Journal of Modelling in Management*, *15*(4), 1419-1450.

Ülkü, M. A., & Gürler, Ü. (2018). The impact of abusing return policies: A newsvendor model with opportunistic consumers. *International Journal of Production Economics*, *203*, 124-133.

Wang, X., Xie, J., & Fan, Z. P. (2021). B2C cross-border E-commerce logistics mode selection considering product returns. *International Journal of Production Research*, *59*(13), 3841-3860.

Webster, S., & Mitra S. (2007). Competitive strategy in remanufacturing and the impact of take-back laws. *Journal of Operations Management*, *25*(6), 1123-1140.

Xu, J., Zhou X., Zhang J., & Long D. Z. (2021). The optimal channel structure with retail costs in a dual-channel supply chain. *International Journal of Production Research*, *59*(1), 47-75.

Ye, G., Hudders L., Jans S. D., & Veirman M.D. (2021). The value of influencer marketing for business: a bibliometric analysis and managerial implications. *Journal of Advertising*, *50*(2), 160-178.

Yoo, S. H., & Cheong, T. (2018). Quality improvement incentive strategies in a supply chain. *Transportation Research Part E: Logistics and Transportation Review*, *114*, 331-342.

Zhang, J., Cao Q., & He X. (2019). Contract and product quality in platform selling. *European Journal of Operational Research*, *272*(3), 928-944.

Zhang, J., Choi, T. M., & Cai, Y. J. (2022). Consumer-to-consumer product trading with strategic consumer behaviors in the sharing economy. *IEEE Transactions on Engineering Management*, *70*(5), 1708-1723.

Zhang, M., Qin F., Wang G. A., & Luo C. (2020). The impact of live video streaming on online purchase intention. *Service Industries Journal*, 40 (9-10), 656-681.

Zhang, X., Chen, H., Hu, J., Ma, C., & Shi, W. (2021). Optimal showroom service strategy and power structure for retailers considering consumer return. *Discrete Dynamics in Nature and Society*, *2021*, 1-22.

Zhang, X., Chen, H., & Liu, Z. (2024). Operation strategy in an E-commerce platform supply chain: whether and how to introduce live streaming services?. *International Transactions in Operational Research*, *31*(2), 1093-1121.

Zhao, S., Wu, F., Jia, T., & Shu, L. (2018). The impact of product returns on price and delivery time competition in online retailing. *Computers & Industrial Engineering*, *125*, 658-667.

Zheng, R., Li, Z., & Na, S. (2022). How customer engagement in the live-streaming affects purchase intention and customer acquisition, E-tailer's perspective. *Journal of Retailing and Consumer Services*, *68*, 103015.

Zhou, Z., Ren, J., Liu, X., & Pardalos, P. M. (2022). Collaborating product development and maintenance service: Quality incentive with inspection and warranty strategies. *International Journal of Production Research*, 1-18.

Zhu, T., Lu, Y., Wang, B., & Zhao, L. (2023). Pre-purchase sales promotions on e-commerce platforms: the effects of promotional benefits on customer-based brand equity. *Journal of Electronic Commerce Research*, *24*(2), 146-170.

APPENDIX

Proof of Theorem 1

We need to first analyze the convexity of Π_L through the first-order and second-order derivatives of Π_L with respect to e. As $\frac{\partial^2 \Pi_L^Y}{\partial e^2} = -2\xi < 0$, the influencer's profit function is concave in the marketing effort and has a maximum value. To obtain the maximum marketing effort, we equate the first-order derivative of the influencer's profit to 0, i.e., $\frac{\partial \Pi_L^Y}{\partial e} = -2e\xi + \frac{Ap\theta\mu\rho}{\tau} = 0$, yielding

$$e^Y = \frac{A\theta\mu\rho p}{2\xi\tau}.$$

Then, substituting e^Y into Eq. (3), as $\frac{\partial^2 \Pi_M}{\partial q^2} = -2\sigma < 0$, we derive the optimal solutions of the brand manufacturer by solving the following equation

$$\frac{\partial \Pi_M^Y}{\partial q} = \frac{Ap\lambda(1-\rho) - 2q\sigma\tau(1-\phi) - A\lambda(c_0 - c_1)}{\tau} = 0.$$

We then find the optimal solutions q^{Y*} and e^{Y*}.

Proof of Proposition 1

q^{Y*}, e^{Y*}, D^{Y*}, R^{Y*}, $D^{Y*} - R^{Y*}$, Π_M^{Y*}, and Π_L^{Y*} must be positive. Letting $D^{Y*} > 0$ and $q^{Y*} > 0$, we get $0 < r < \hat{r}_2$ and $0 < \rho < \frac{p - c_0 + c_1}{p}$. Especially, when $\hat{r}_1 < r < \hat{r}_2$, $R^{Y*} = 0$. Then, we find the feasible ranges for the profits by judging the monotonicity of the profits with respect to ϕ.

$$\frac{\partial \Pi_L^{Y*}}{\partial \phi} = \frac{A^2\lambda^2(-p(1-\rho) + c_0 - c_1)(p(1-3\rho) - c_0 + c_1 + \phi(p + p\rho - c_0 + c_1))}{4\sigma\tau^2(1-\phi)^3},$$

$$\frac{\partial \Pi_M^{Y*}}{\partial \phi} = \frac{A^2\lambda^2(p(1-\rho) - c_0 + c_1)^2}{4\sigma\tau^2(1-\phi)^2} > 0,$$

$$\lim_{\phi \to 1} \Pi_L^{Y*} = -\infty,$$

$$\lim_{\phi \to 0} \Pi_L^{Y*} = F + \frac{A\begin{pmatrix} p\left(Ap\left(2\lambda^2\xi(1-\rho)\rho + \theta^2\mu^2\rho^2\sigma\right) - 4\xi\rho\sigma(r + p\alpha - \tau)\tau\right) \\ -2Ap\lambda^2\xi\rho(c_0 - c_1) \end{pmatrix}}{4\xi\sigma\tau^2},$$

$$\lim_{\phi \to 1} \Pi_M^{Y*} = +\infty,$$

$$\lim_{\phi \to 0} \Pi_M^{Y*} = -F + \frac{A\begin{pmatrix} (-p(1-\rho) + c_0)\begin{pmatrix} -Ap\lambda^2\xi(1-\rho) - 2p\theta^2\mu^2\rho\sigma \\ +4(r + p\alpha)\xi\sigma\tau - 4\xi\sigma\tau^2 + A\lambda^2\xi c_0 \end{pmatrix} \\ -2\begin{pmatrix} -Ap\lambda^2\xi(1-\rho) - 2\xi\sigma\tau(r + r\tau - Q\lambda\tau) \\ +p\sigma(1-\tau)(2\alpha\xi\tau - \theta^2\mu^2\rho) + A\lambda^2\xi c_0 \end{pmatrix} c_1 + A\lambda^2\xi c_1^2 \end{pmatrix}}{4\xi\sigma\tau^2}.$$

(1) Analysis of Π_L^{Y*}:

When $0 < \rho < \frac{p - c_0 + c_1}{3p}$, $\frac{\partial \Pi_L^{Y*}}{\partial \phi} < 0$.

When $\frac{p - c_0 + c_1}{3p} < \rho < \frac{p - c_0 + c_1}{p}$ and $\phi > \frac{(3\rho - 1) + c_0 - c_1}{p(1+\rho) - c_0 + c_1}$, $\frac{\partial \Pi_L^{Y*}}{\partial \phi} < 0$.

When $\frac{p - c_0 + c_1}{3p} < \rho < \frac{p - c_0 + c_1}{p}$ and $\phi < \frac{(3\rho - 1) + c_0 - c_1}{p(1+\rho) - c_0 + c_1}$, $\frac{\partial \Pi_L^{Y*}}{\partial \phi} > 0$.

So, when $0 < \rho < \frac{p - c_0 + c_1}{3p}$ and $\lim_{\phi \to 0} \Pi_L^{Y*} > 0$, letting $\Pi_L^{Y*} = \frac{B_1\phi^2 + B_2\phi + B_3}{4\xi\sigma\tau^2(1-\phi)^2} = 0$, we get $\phi_1 = \frac{-B_2 + \sqrt{B_2^2 - 4B_1B_3}}{2B_1}$ and $\phi_2 = \frac{-B_2 - \sqrt{B_2^2 - 4B_1B_3}}{2B_1}$. Thus, when $0 < \rho < \frac{p - c_0 + c_1}{3p}$, $F > \hat{F}_1$, and $0 \le \phi < \phi_1$, $\Pi_L^{Y*} > 0$.

When $\frac{p - c_0 + c_1}{3p} < \rho < \frac{p - c_0 + c_1}{p}$ and $\Pi_L^{Y*}\left(\phi = \frac{(3\rho - 1) + c_0 - c_1}{p(1+\rho) - c_0 + c_1}\right) = 0$, letting $\Pi_L^{Y*} > 0$, we get $F > \hat{F}_2$. Thus, when ① $\frac{p - c_0 + c_1}{3p} < \rho < \frac{p - c_0 + c_1}{p}$, $F > \hat{F}_1$, and $0 \le \phi < \phi_1$ or ② $\frac{p - c_0 + c_1}{3p} < \rho < \frac{p - c_0 + c_1}{p}$, $\hat{F}_2 < F < \hat{F}_1$, and $\phi_2 < \phi < \phi_1$, $\Pi_L^{Y*} > 0$, so $\hat{F}_1 > \hat{F}_2$.

(2) Analysis of Π_M^{Y*}:

When $\lim_{\phi \to 0} \Pi_M^{Y*} > 0$, i.e., $F < \hat{F}_3$, $\Pi_M^{Y*} > 0$. When $\lim_{\phi \to 0} \Pi_M^{Y*} < 0$, i.e., $F > \hat{F}_3$, letting $\Pi_M^{Y*} = 0$, we get

$$\phi_3 = \frac{\begin{pmatrix} -A^2p^2\lambda^2\xi(1-\rho)^2+4F\xi\sigma\tau^2-2Ap(1-\rho)\sigma\left(p\theta^2\mu^2\rho-2(r+p\alpha)\xi\tau+2\xi\tau^2\right) \\ +A\begin{pmatrix} +c_0\left(2Ap\lambda^2\xi(1-\rho)+2p\theta^2\mu^2\rho\sigma+4(r+p\alpha)\xi\sigma\tau-4\xi\sigma\tau^2+A\lambda^2\xi c_0\right) \\ +2\left(-Ap\lambda^2\xi(1-\rho)-2\xi\sigma\tau(r+r\tau-Q\lambda\tau)+p\sigma(1-\tau)\left(2\alpha\xi\tau-\theta^2\mu^2\rho\right)+A\lambda^2\xi c_0\right)c_1-A\lambda^2\xi c_1^2 \end{pmatrix} \end{pmatrix}}{2\sigma\begin{pmatrix} 2F\xi\tau^2-A(p\theta^2\mu^2\rho-2(r+p\alpha)\xi\tau+2\xi\tau^2)(p-p\rho-c_0) \\ -A(2\xi\tau(r+r\tau-Q\lambda\tau)+p(1-\tau)(\theta^2\mu^2\rho-2\alpha\xi\tau))c_1 \end{pmatrix}}.$$

Thus, when $F > \hat{F}_3$ and $\phi > \phi_3$, $\Pi_M^{Y*} > 0$, where

$\hat{F}_1 = \frac{A\left(p\left(Ap\left(2\lambda^2\xi(1-\rho)\rho-\theta^2\mu^2\rho^2\sigma\right)+4\xi\rho\sigma(r+p\alpha-\tau)\tau\right)+2Ap\lambda^2\xi\rho(c_0-c_1)\right)}{4\xi\sigma\tau^2}$,

$\hat{F}_2 = \frac{A\left(p\left(-Ap\left(\lambda^2\xi(1+\rho)^2+4\theta^2\mu^2\rho^2\sigma\right)+16\xi\rho\sigma(r+p\alpha-\tau)\tau\right)+A\lambda^2\xi(c_0-c_1)(2p(1+\rho)-c_0+c_1)\right)}{16\xi\sigma\tau^2}$,

$\hat{F}_3 = \frac{A\begin{pmatrix} (-p(1-\rho)+c_0)\left(-Ap\lambda^2\xi(1-\rho)-2p\theta^2\mu^2\rho\sigma+4(r+p\alpha)\xi\sigma\tau-4\xi\sigma\tau^2+A\lambda^2\xi c_0\right) \\ -2\left(-Ap\lambda^2\xi(1-\rho)-2\xi\sigma\tau(r+r\tau-Q\lambda\tau)+p\sigma(-1+\tau)(\theta^2\mu^2\rho-2\alpha\xi\tau)+A\lambda^2\xi c_0\right)c_1+A\lambda^2\xi c_1^2 \end{pmatrix}}{4\xi\sigma\tau^2}$,

$\hat{r}_1 = \frac{\left(-Ap\left(\lambda^2\xi(1-\rho)-\theta^2\mu^2\rho\sigma(1-\tau)(1-\phi)\right)+2\xi\sigma\tau(p\alpha(1-\tau)+Q\lambda\tau)(1-\phi)+A\lambda^2\xi(c_0-c_1)\right)}{2\xi\sigma\tau(1+\tau)(1-\phi)}$,

$\hat{r}_2 = \frac{Ap\theta^2\mu^2\rho+2(1-p\alpha+Q\lambda)\xi\tau}{2\xi\tau}$.

Proof of Proposition 2

Sensitivity analysis of λ without quality cooperation:

$\frac{\partial e^{N*}}{\partial\lambda} = 0$,

$\frac{\partial q^{N*}}{\partial\lambda} = \frac{A(p-p\rho-c_0+c_1)}{2\sigma\tau(1-\phi)} > 0$,

$\frac{\partial(D^{N*}-R^{N*})}{\partial\lambda} = \frac{A^2\lambda(p(1-\rho)-c_0+c_1)}{\sigma\tau^2} > 0$,

$\frac{\partial\Pi_L^{N*}}{\partial\lambda} = \frac{A^2p\lambda\rho(p(1-\rho)-c_0+c_1)}{\sigma\tau^2} > 0$,

$\frac{\partial\Pi_M^{N*}}{\partial\lambda} = \frac{A(A\lambda(p(1-\rho)-c_0+c_1)^2-2Q\sigma\tau^2c_1)}{2\sigma\tau^2}$.

When $A\lambda(p(1-\rho)-c_0+c_1)^2-2Q\sigma\tau^2c_1 > 0$, i.e., $\sigma < \frac{A\lambda(p(1-\rho)-c_0+c_1)^2}{2Q\tau^2c_1}$ and $\frac{\partial\Pi_M^{N*}}{\partial\lambda} > 0$; otherwise, $\frac{\partial\Pi_M^{N*}}{\partial\lambda} < 0$.

Sensitivity analysis of λ with quality cooperation:

$\frac{\partial e^{Y*}}{\partial\lambda} = 0$,

$\frac{\partial q^{Y*}}{\partial\lambda} = \frac{A(p(1-\rho)-c_0+c_1)}{2\sigma\tau} > 0$,

$\frac{\partial(D^{Y*}-R^{Y*})}{\partial\lambda} = \frac{A^2\lambda(p(1-\rho)-c_0+c_1)}{\sigma\tau^2(1-\phi)} > 0$,

$\frac{\partial\Pi_L^{Y*}}{\partial\lambda} = \frac{-A^2\lambda(p(1-\rho)-c_0+c_1)(p(\rho(-2+\phi)+\phi)-\phi c_0+\phi c_1)}{2\sigma\tau^2(1-\phi)^2}$.

When $p(\rho(-2+\phi)+\phi)-\phi(c_0-c_1) > 0$, i.e., $\rho < \frac{\phi(p-c_0+c_1)}{p(2-\phi)}$ and $\frac{\partial\Pi_L^{Y*}}{\partial\lambda} < 0$; otherwise, $\frac{\partial\Pi_L^{Y*}}{\partial\lambda} > 0$.

$\frac{\partial\Pi_M^{Y*}}{\partial\lambda} = \frac{A\left(-A\lambda(p(1-\rho)-c_0)^2-A\lambda c_1^2+2(-Ap\lambda(1-\rho)+Q\sigma\tau^2(1-\phi)+A\lambda c_0)c_1\right)}{2\sigma\tau^2(-1+\phi)}$.

When $-A\lambda(p(1-\rho)-c_0)^2-2(Ap\lambda(1-\rho)+Q\sigma\tau^2(1-\phi)+A\lambda c_0)c_1-A\lambda c_1^2 > 0$, i.e., $\sigma < \frac{A\lambda(p(1-\rho)-c_0+c_1)^2}{2Q\tau^2(1-\phi)c_1}$, $\frac{\partial\Pi_M^{Y*}}{\partial\lambda} > 0$; otherwise, $\frac{\partial\Pi_M^{Y*}}{\partial\lambda} < 0$.

Proof of Proposition 3

Sensitivity analysis of θ without quality cooperation:

$\frac{\partial e^{N*}}{\partial\theta} = \frac{\mu\rho p}{2\xi\tau} > 0$,

$\frac{\partial q^{N*}}{\partial\theta} = 0$,

$\frac{\partial(D^{N*}-R^{N*})}{\partial\theta} = \frac{Ap\theta\mu^2\rho}{\xi\tau^2} > 0$,

$\frac{\partial\Pi_L^{N*}}{\partial\theta} = \frac{(2A-1)p^2\theta\mu^2\rho^2}{2\xi\tau^2}$.

When A >0.5, $\frac{\partial\Pi_L^{N*}}{\partial\theta} > 0$; otherwise, $\frac{\partial\Pi_L^{N*}}{\partial\theta} < 0$.

$\frac{\partial \Pi_M^{N*}}{\partial \theta} = \frac{Ap\theta\mu^2\rho(p(1-\rho)-c_0+(1-\tau)c_1)}{\xi\tau^2}$.

When $p(1-\rho)-c_0+(1-\tau)c_1 > 0$, i.e., $\rho < 1 - \frac{c_0-(1-\tau)c_1}{p}$ and $\frac{\partial \Pi_M^{N*}}{\partial \theta} > 0$; otherwise, $\frac{\partial \Pi_M^{N*}}{\partial \theta} < 0$.

Sensitivity analysis of θ with quality cooperation is the same as the situation with quality cooperation.

Proof of Proposition 4

Comparison of quality:

$q^{N*} - q^{Y*} = \frac{A\lambda(p(1-\rho)-c_0+c_1)}{2\sigma\tau} - \frac{A\lambda(p(1-\rho)-c_0+c_1)}{2\sigma\tau(1-\phi)} = -\frac{A\lambda\phi(p(1-\rho)-c_0+c_1)}{2\sigma\tau(1-\phi)} < 0.$

Comparison of the marketing effort level:

$e^{N*} - e^{Y*} = \frac{\theta\mu\rho}{2\xi\tau}p - \frac{\theta\mu\rho}{2\xi\tau}p = 0.$

Comparison of demand:

$D^{N*} - D^{Y*} = \theta\mu(e^{N*} - e^{Y*}) = 0.$

Comparison of return volume:

$R^{N*} - R^{Y*} = \frac{A\left(\theta\mu(-1+\tau)(e^{N*}-e^{Y*})-\lambda(q^{N*}-q^{Y*})\right)}{\tau} = \frac{-\lambda(q^{N*}-q^{Y*})}{\tau} > 0.$

Comparison of actual sales:

$(D^{N*} - R^{N*}) - (D^{Y*} - R^{Y*}) = (D^{N*} - D^{Y*}) - (R^{N*} - R^{Y*}) = -(R^{N*} - R^{Y*}) < 0.$

Comparison of return rate:

$\frac{R^{N*}}{D^{N*}} - \frac{R^{Y*}}{D^{Y*}} = \frac{R^{N*}D^{Y*}-D^{N*}R^{Y*}}{D^{N*}D^{Y*}} = \frac{R^{N*}(D^{Y*}-R^{Y*})-(D^{N*}-R^{N*})R^{Y*}}{D^{N*}D^{Y*}} > 0.$

Proof of Proposition 5

Comparison of the influencer's profit:

$\Pi_L^{N*} - \Pi_L^{Y*} = \frac{A^2\lambda^2\phi(p(1-\rho)-c_0+c_1)(p+p\rho(-3+2\phi)-c_0+c_1)}{4\sigma\tau^2(-1+\phi)^2}$.

When $0 < \rho < \frac{p-c_0+c_1}{3p}$, $\Pi_L^{N*} > \Pi_L^{Y*}$. When $\frac{p-c_0+c_1}{3p} < \rho < \frac{p-c_0+c_1}{p}$, if $0 < \phi < \frac{p(3\rho-1)++c_0-c_1}{2p\rho}$, $\Pi_L^{N*} < \Pi_L^{Y*}$; otherwise, $\Pi_L^{N*} > \Pi_L^{Y*}$.

Comparison of the brand manufacturer's profit:

$\Pi_M^{N*} - \Pi_M^{Y*} = -\frac{A^2\lambda^2\phi(p(1-\rho)-c_0+c_1)^2}{4\sigma\tau^2(1-\phi)} < 0.$

Proof of Proposition 6

$q_1{}^{Y*} - q_2{}^{Y*} = \frac{A\beta\lambda(p-p\rho-c_0+c_1)}{2\sigma\tau} - \frac{Ap\beta\lambda\rho}{2\sigma\tau} = \frac{A\beta\lambda(p-2p\rho-c_0+c_1)}{2\sigma\tau}$

$e^{N*} - e^{Y*} = \frac{\theta\mu\rho}{2\xi\tau}p - \frac{\theta\mu\rho}{2\xi\tau}p = 0$

$q^{N*} - q^{Y*} = \frac{A\lambda(p(1-\rho)-c_0+c_1)}{2\sigma\tau} - \left(\frac{A\lambda(p-p\rho-c_0+c_1)}{2\sigma\tau} + \frac{Ap\lambda\rho}{2\sigma\tau}\right) = -\frac{Ap\lambda\rho}{2\sigma\tau} < 0$

$D^{N*} - D^{Y*} = \theta\mu(e^{N*} - e^{Y*}) = 0$

$R^{N*} - R^{Y*} = \frac{A(\theta\mu(-1+\tau)(e^{N*}-e^{Y*})-\lambda(q^{N*}-q^{Y*}))}{\tau} = \frac{-\lambda(q^{N*}-q^{Y*})}{\tau} > 0$

$(D^{N*} - R^{N*}) - (D^{Y*} - R^{Y*}) = (D^{N*} - D^{Y*}) - (R^{N*} - R^{Y*}) = -(R^{N*} - R^{Y*}) < 0$

$\frac{R^{N*}}{D^{N*}} - \frac{R^{Y*}}{D^{Y*}} = \frac{R^{N*}D^{Y*}-D^{N*}R^{Y*}}{D^{N*}D^{Y*}} = \frac{R^{N*}D^{Y*}-R^{N*}R^{Y*}+R^{N*}R^{Y*}-D^{N*}R^{Y*}}{D^{N*}D^{Y*}} = \frac{R^{N*}(D^{Y*}-R^{Y*})-(D^{N*}-R^{N*})R^{Y*}}{D^{N*}D^{Y*}} > 0$

$\pi_M^{N*} - \pi_M^{Y*} = \frac{A^2p\lambda^2\rho(p-p\rho-c_0+c_1)}{2\sigma\tau^2} > 0$

$\pi_L^{N*} - \pi_L^{Y*} = -\frac{A^2p^2\lambda^2\rho^2}{4\sigma\tau^2} < 0$